THE FUTURE...

- discrete univariate random variables
- multivariate distributions
- time series analysis
- stochastic processes

SOME *APPL*ICATIONS

- Determine the exact distribution of $\bar{X}$ in the presence of skewed data or outliers to make statistical inferences about the population mean

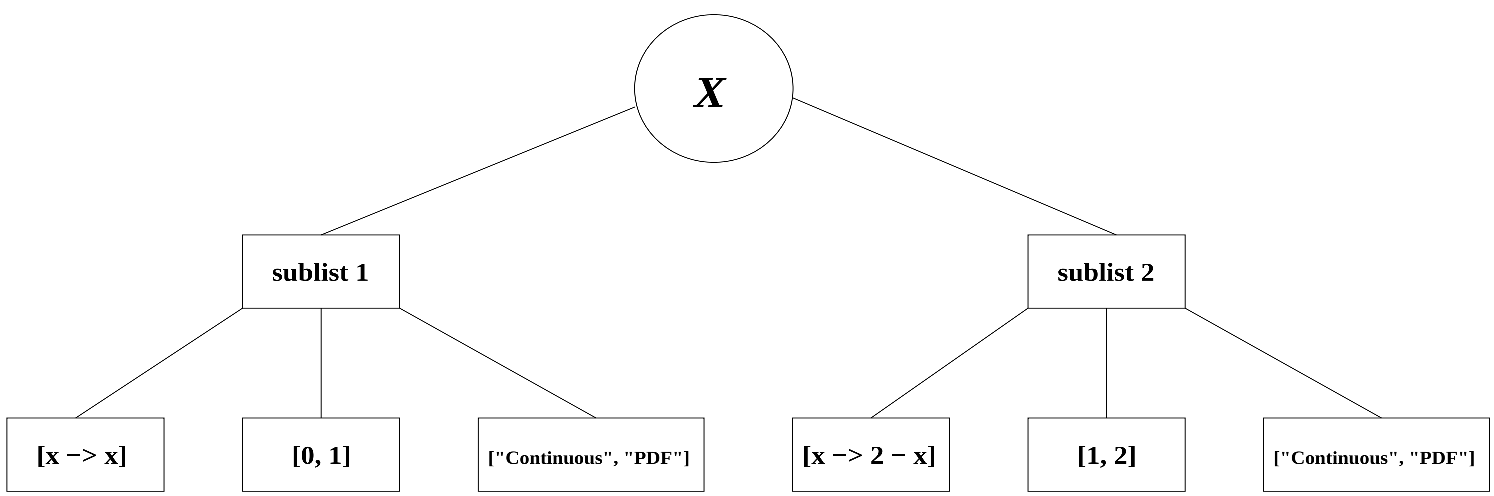
- Generate *resources* such as tables, graphs, and charts which include means, critical points, variances, etc

- Modeling using the hazard function

- Identification of outliers in a dataset

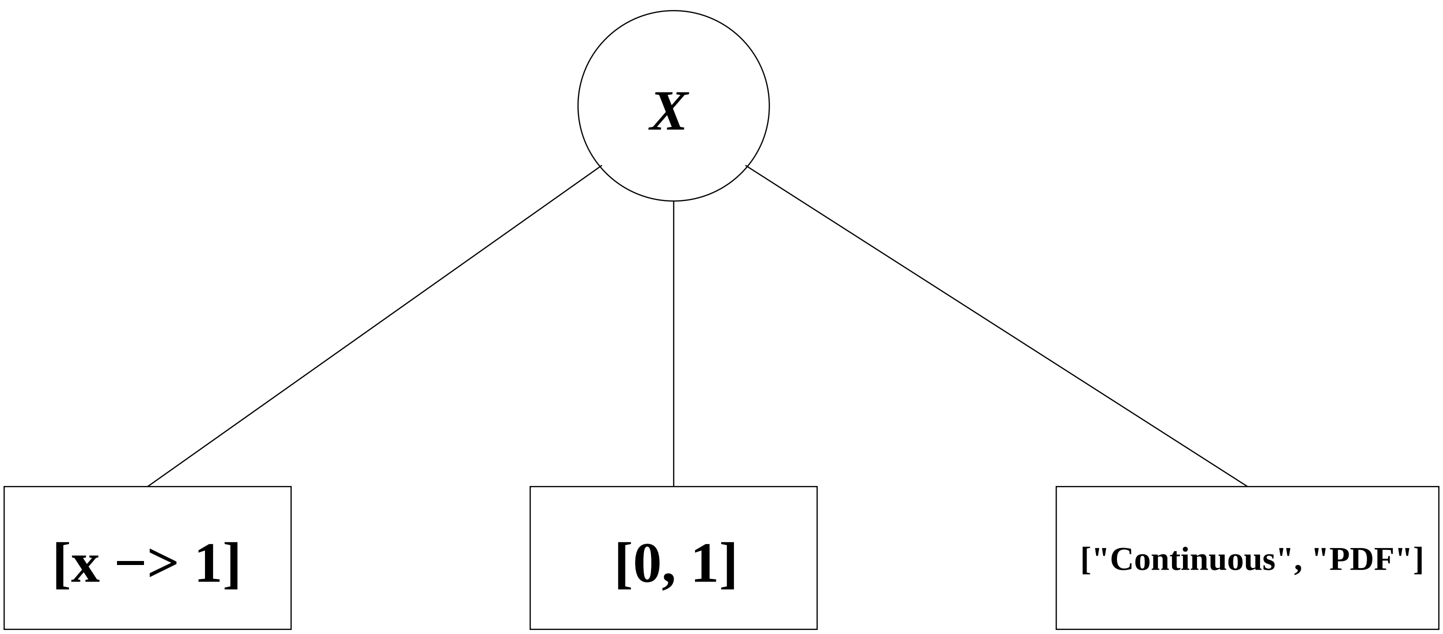
EXAMPLES (cont'd)

```
X := [[x -> x, x -> 2-x], [0, 1, 2],
      ["Continuous", "PDF"]];
```



EXAMPLES

```
X := [[x -> 1], [0, 1], ["Continuous",
"PDF"]];
```



LIST OF SUBLISTS

- ordered functions that define the functional forms of distribution

- ordered list of real numbers defining end-points of intervals in the first sublist

- indicates what distribution form the functions in the first sublist represent

THE COMMON DATA STRUCTURE

- ONE data structure that applies to all six functional forms

- referred to as the "list of sublists"

- each random variable is presented in a list containing three sublists

MORE APPL ALGORITHMS

- generate random variates associated with a random variable
- plot the six functional forms of any fully-specified distribution
- provide basic statistical abilities
- determine common summary characteristics of random variates - mean, variance, etc

MORE APPL ALGORITHMS

- calculate the PDF of sums of independent random variables: $Z = X + Y$

- calculate the PDF of products of independent random variables: $Z = XY$

- calculate the PDF of the min and the max of independent random variables: $Z = min\{X, Y\}$ and $Z = max\{X, Y\}$

- calculate the PDF of the rth order statistic from a sample of n iid random variables

APPL ALGORITHMS

- supply a common data structure for representing the distributions of univariate random variables

- convert amonsgt six functional forms (PDF, CDF, IDF, SF, HF, CHF)

- provide straightforward instantiation of well-known distributions (exponential, normal, uniform)

- determine the distribution of a simple transformation of a continuous random variable, $Y = g(x)$

APPL Overview

- MAPLE statistical procedures cannot be applied to PDFs - obtaining probability results impossible

- Karian and Tanis supplemented MAPLE with "about 130 procedures written specifically to promote explorations of probabalistic and statistical concepts"

- focus of APPL is constructing a probability package for random variables

- APPL is limited to univariate, continuous, independent random variables and the transformations and combinations that result

PROBABILITY TERMS

- *piecewise:* functions constructed by piecing together various functions

- *iid:* independent and identically distributed random variables

- specification:
  - *fully specified:* `Uniform(0, 1)`
  - *semi-specified:* `Uniform(a, 1)`
  - *unspecified:* `Uniform(a, b)`

MORE PROBABILITY FUNCTIONS

- Survivor Function (SF):

  - $S_X(x) = 1 - F_X(x)$

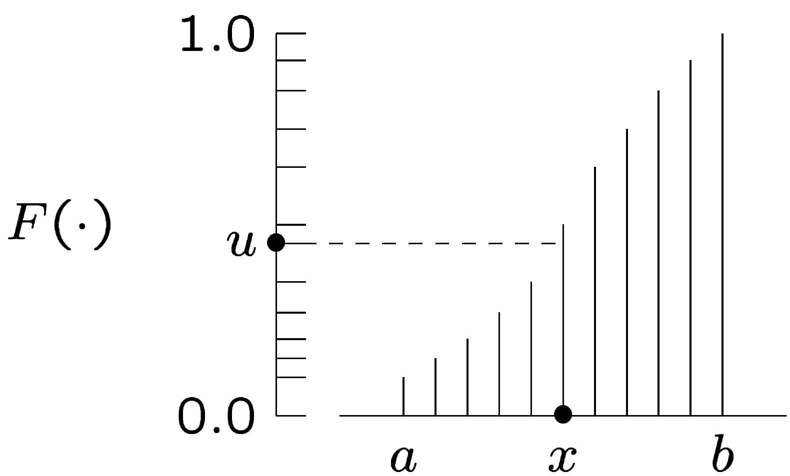
- Hazard Function (HF):

  - $h_X(x) = \frac{f_X(x)}{S_X(x)}$

- Cumulative Hazard Function (CHF):

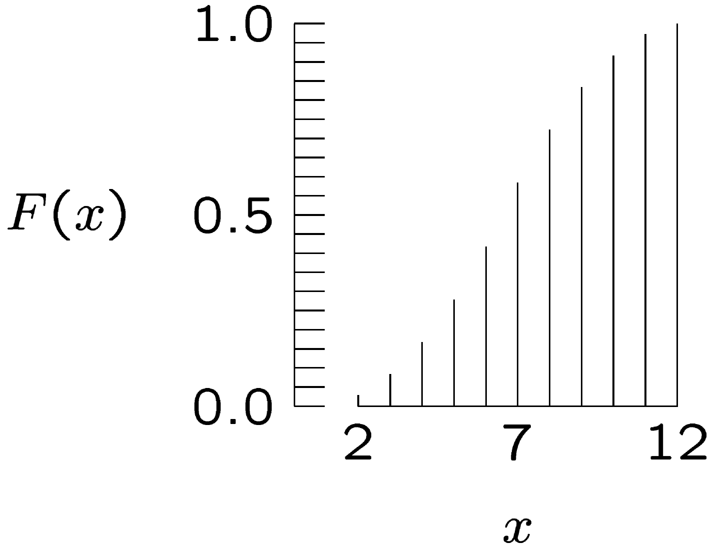
  - $H_X(x) = \int_{-\infty}^{x} h_X(s)ds$

Inverse Distribution Function (IDF)



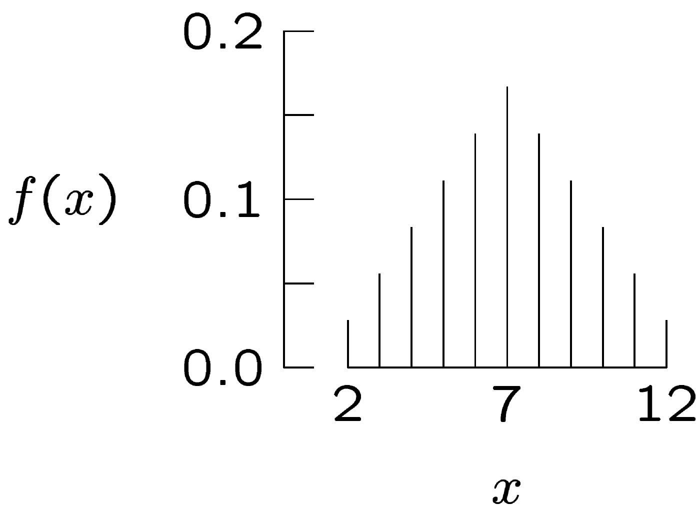
$$F_X^{-1}(u) = \min_x \{x : u < F(x)\}$$

Cumulative Distribution Function (CDF)



$$F_X(x) = \int_{-\infty}^{x} f_X(s)\, ds$$

Probability Density Function (PDF)



$$f_X(x) = \Pr(X = x)$$

APPL: BACKGROUND

- MAPLE based conceptual probability software package

- fills existing technological gap in probability theory

- successfully integrated into three classes at W&M

INTRODUCTION

- Axioms/Theorems:
  - name and describe frequent random variables
  - provide theoretical results of random variables
  - provide applied results for statistics applications
- Omission:
  - ability to automate naming, processing, and application of random variables (very tedious)

APPL: A Probability Programming Language

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