Monte Carlo Method Samples

-By combining renormaliztion methods and Monte Carlo techniques for making b very large it is possible to generate $\nu \sim \frac{4}{3}$ and generate today's best known estimate for $p_c = .59827$

Sample Calculation

$\nu = \frac{log2}{log1.5274} = 1.635$

ν**exact** $= \frac{4}{3}$

-"interface" problem

-reduced as b gets large

More Renormalizing

p' - p* = R(p) - R(p*) $\sim \lambda(p - p*)$

$$\lambda = \frac{dR}{dp}|p = p*$$

$$|p' - p * |^{-\nu} = lambda^{-\nu}|p - p * |^{-\nu}$$

from last slide:

$$b^{-1} = \lambda^{-\nu}$$

$$\nu = \frac{logb}{lag\lambda}$$

Application of Renormalization

-Solving the recursion relation for b=2

p*=.61804

-The renormalized connectedness lengths

$\xi' = \frac{\xi}{b}$

-because all lengths contract by distance b

-Since $\xi(p) = const|p - p_c|^{-\nu} for p \sim p_c$

$|p' - p*|^{-\nu} = b^{-1}|p - p*|^{-\nu}$

Renormalized Derivations

define p' = R(p)

p' is the probability that a renormalized square is occupied

p' = sum of all possiblities p' = R(p) = $p^4 + 4p^3(1-p) + 2p^2(1-p)^2$

-notice that if p0=.5, p1=.44, p2=.35 $\ldots pn \to 0$

at p_c we want p* = R(p*)

Renormaliztion Group

Dimension Dependence

-Earlier, p_c depended on symmetry and dimension of lattice

-Critical Exponents don't explicity depend on dimension

-Introduce Scaling Law

$$2\beta + \gamma = \nu d$$

-Derivation of law can be found in references

More Finite Size Scaling

Since $\xi(p) \sim$ L

$$P_\infty(p = p_c) \sim L^{\frac{-\beta}{\nu}} (L \to \infty)$$

-use to evaluate critical exponents

-generate p = p_c for increasing values of L

-calculate P_∞ as a function of L

-as L gets large, estimate $\frac{\beta}{\nu}$

Fininte Size Scaling

In critical region

$\xi(p) \sim L \sim |p - p_c|^{-\nu}$

or $|p - p_c| \sim L^{\frac{-1}{\nu}}$

Correcting for the Divergence

-Results obtained are close to table

-Differnece is cause by finite size of grid and the divergence of ξ

-For $p \ll p_c$ and $p \gg p_c$ the system is indistinguishable from a macroscopic system

Redefining P_∞

in critical region

$P_\infty \sim (p_c - p)^\beta$

P_∞ is called the order parameter of the model

β describes how connectedness of the infinte cluster goes to 0 at threshold

also, mean cluster size S(P)

$\mathrm{S(P)} \sim |p - p_c|^{-\gamma}$

Divergence of $\xi(p)$

Define **critical expnonent** ν such that

-$\xi(p) \sim |p - p_c|^{-\nu}$

-also notice $\xi(p_c) \sim L$ so that

-$\xi(p)$ diverges in the "critical region" $|p - p_c| \ll 1$ as $L \to \infty$

Mean Connectedness Length

$\xi_p(p)$ = Radius of Gyration

$R_s^2 = \frac{1}{s}\sum_i(\mathbf{r_i} - \mathbf{r})^2$

where $\mathbf{r} = \frac{1}{s}\sum_i \mathbf{r_i}$

-Notice $\mathbf{r}$ is the well defined center of mass of cluster

-ξ is associated with the largest non-spanning cluster

Critical Exponents

-Thermodynamical phase transitions

-Curie “critical” temperature

-Percolation phase transition

-All have to do with large finite blocks

-Properties of n_s

- decreases rapidly for $p < p_c$ and $p > p_c$
- decreases slowly for $p = p_c$

Labeling Routine

1. Initialize the Array
2. Define array of size of number of labels, initialize each value to 0
3. Set np[label] = minimum label of neighbors
4. Assign neighbors to minimum proper label
5. Set each occupied site equal to assigned proper labels

Cluster Labeling

-Hoshen and Kopelman

np(0)=0 np(1)=0 np(2)=0 np(3)=0 np(4)=3 np(5)=3

Spanning Infinite Clusters

$$P_\infty(p) = \frac{\textbf{num sites in spanning cluster}}{\textbf{num occupied sites}}$$

For and infinite lattice

$$P_\infty(p < p_c) = 0$$

$$P_\infty(p > p_c) = increasing$$

$$P_\infty(p = 1) = 1$$

Quantities of Interest

mean cluster size distribution

$$\mathrm{ns} = \frac{\textbf{num clusters of size s}}{\textbf{num of items in lattice}}$$

probability site w belogs to s size cluster

$$w_s = \frac{s * n_s}{\sum_s n_s}$$

mean cluster size

$$s = \sum_s s * w_s = \frac{\sum_s s^2 * n_s}{\sum_s s * n_s}$$

(spanning cluster is excluded)

Percolation Threshold

- P_c is the probablity at which an infinte spanning cluster first appears in an infinite lattice. (a.k.a. "percolation phase transition")
- p_c (L) is average value of p_c for various L's and approaches p_c as L appraoches infinity
- p_c (L) depends on the symetry and dimensions of lattice
- p_c (L) depends on the current definition of spanning

Connectedness is Key

-Four percolation probablities

Clustering

Clusters

vs

Spanning Cluster

-Spanning in one direction for convenience

Critical Probability

$$p \geq p_c$$

and

$$p < p_c$$

Types of Percolation

Site

Bond

Continuum

Visualizing Perolation

Cookies

Clusters -Finite Cells

-Universal Local Probablity

Percolation

-foundations of the new physics

Introduction

-physical relations (*phase transition*, *conduction*)

-what is *exactly* is percolation?

-types of percolation

The Percolation Threshold

Implementation Method

-Labeling Routine

Critical Exponents and Finite Size Scaling

Renormalization Methods

-Monte Carlo Applications