Computer Science 616 - Stochastic Models in Computer Science
Fall 2007, Homework 1, due Sept 7 at noon

1. We know that the definition of independence for n events $\mathcal{E} = \{E_1, E_2, \ldots, E_n\}$,

$$\forall \mathcal{F}, \mathcal{F} \subseteq \mathcal{E}, \mathcal{F} = \{F_1, F_2, \ldots, F_m\}, \quad P(F_1 \cap F_2 \cap \ldots \cap F_m) = P(F_1)P(F_2) \cdots P(F_m)$$

is not equivalent to that of pairwise independence,

$$\forall i, j, i \neq j, 1 \leq i, j \leq n, \quad P(E_i \cap E_j) = P(E_i)P(E_j).$$

Show, by example, that is also not equivalent to just requiring that

$$P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1)P(E_2) \cdots P(E_n).$$

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2. (Section 1, Exercise 8) Prove that, for any pair of events E and F,

$$P(E \cap F) \geq P(E) + P(F) - 1.$$

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3. (Section 1, Exercise 23) Given n events $E_1, E_2, \ldots, E_n$, show that

$$P(E_1 \cap \cdots \cap E_n) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_1 \cap E_2) \cdots P(E_n|E_1 \cap \cdots \cap E_{n-1}).$$

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4. (Section 1, Exercise 39) Stores A, B, and C have 50, 75, and 100 employees, and respectively 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store C? 10

5. (Section 2, Exercises 4 and 5) Suppose a die is rolled twice. What are the possible values that the following random variable can assume?

   (a) The maximum value to appear in any of the two rolls.

   (b) The minimum value to appear in any of the two rolls.

   (c) The sum of the two rolls.

   (d) The difference between the first and the second roll.

   Assuming that the die is fair, compute the probabilities associated with the above four random variables. 60

6. Throw two dice. What is the conditional probability that the first one is a 6 given that the sum is 8? 10

7. Urn 1 has 10 black balls and 8 white balls. Urn 2 has 5 black balls and 3 white balls. You flip a fair coin and select a ball from urn 1 if the outcome is heads, or from urn 2 if the outcome is tails. What is the probability that the coin landed heads if you selected a black ball? 20