Computer Science 616 - Stochastic Models in Computer Science
Fall 2007, Homework 2, due Sept 17 at noon

1. (Section 2, Exercise 12) On an exam with five multiple-choice questions, each with three possible answers, what is the probability of getting four or more correct answers by guessing? 10

2. (Section 2, Exercise 30) Assume that $X \sim Poisson(\lambda)$. Show that, for $i \geq 0$, $p_X(i)$ grows monotonically as a function of i until i reaches $\lfloor \lambda \rfloor$, then it decreases monotonically after that. 20

3. What is the distribution (pmf) and the expectation of the random variable X representing the *algebraic difference* (i.e., the value of X value can be negative) between the number of heads and tails obtained in n tosses, if p is the probability of the coin coming up heads? 20

4. Assume that n independent experiments are performed, and that, for each of them, the outcome is $i = 1, 2, \ldots, r$ with probability $p_1, p_2, \ldots, p_r$, respectively (thus, $\sum_{i=1}^{r} p_i = 1$). What is the probability that exactly $x_1, x_2, \ldots, x_r$ outcomes occur of each type (with $x_1+x_2+\cdots x_r = n$)? Provide a function for the pmf, argue why the function need to be as you define it, and finally provide a formal proof that the function is indeed a probability distribution.
   Hint: high dimensional problems have small dimensional special cases that may be helpful for a better understanding and to recognize how one may reach the general case. 50