Computer Science 616 - Stochastic Models in Computer Science
Fall 2007, Homework 5, due Oct 31 at noon

1. (Section 3, #24) A coin having probability p of landing heads is continually flipped until at least one head and one tail have been flipped.
   (a) Find the expected number of flips needed.
   (b) Find the expected number of flips that lands on heads.
   (c) Find the expected number of flips that lands on tails.
   (d) Find the expected number of flips needed until there has been a total of at least two heads and one tail. 50

2. A round of Lose Your Money Fast (LYMF) allows you to bet any non-negative amount $\$x$ and returns to you $100 \times \$x$ with probability 0.001, $20 \times \$x$ with probability 0.005, and nothing otherwise. If you start with \$10 and keep playing all you have (including your winnings, if any) at each round, how much money do you expect to have after n rounds of LYMF? 30

3. (Section 3, #22) A coin having probability p of coming up heads is successively flipped until two of the most recent three flips are heads. Let N denote the number of flips (if the first two flips are heads, $N = 2$). Compute $E[N]$. 30

4. Consider a "car parking problem" where you have a side of a city block of length b, along which cars of length c can park. Starting from an empty block, any arriving car can park in any position of any empty space of length c or greater. We are intersted in computing the expected number of cars that will fit (that is, the expected number of cars that can find a place to park, until all remaining empty spaces are of size less than c). To simplify the problem, assume that b and c are positive integers and that cars can park only at integral positions (i.e., the first car can park at position $0, 1, \ldots, b - c$), with uniform probability (of course, only the case $b \geq c$ is interesting and the case $c = 1$ is trivial).
   - Find a recursive expression for the expected number of cars that can fit.
   - Compute the expected number of cars that can fit for $c = 2$ and b up to 6 (please use fractions, not decimals, in your computations). 30

5. Alice, Boris, and Carlos are three evenly matched tennis players. Initially, Alice and Boris play a set, and the winner plays Carlos. This continues, with the winner of a set always playing the waiting player, until one of the players has won two sets in a row. That player is then declared the overall winner. Find the probability that Alice is the overall winner. 40