

# Estimating Surface Normals from Spherical Stokes Reflectance Fields

## Introduction

In this work, we propose a novel method for estimating surface orientation from the Stokes polarization vector of specularly reflected light under a single spherical incident illumination condition that is either circularly polarized (Fig. 1, c-d) or unpolarized (Fig. 1, e-f). Polarization cues have previously been employed to separate diffuse and specular reflectance components, to classify materials, to estimate reflectance properties, and to estimate surface normals.

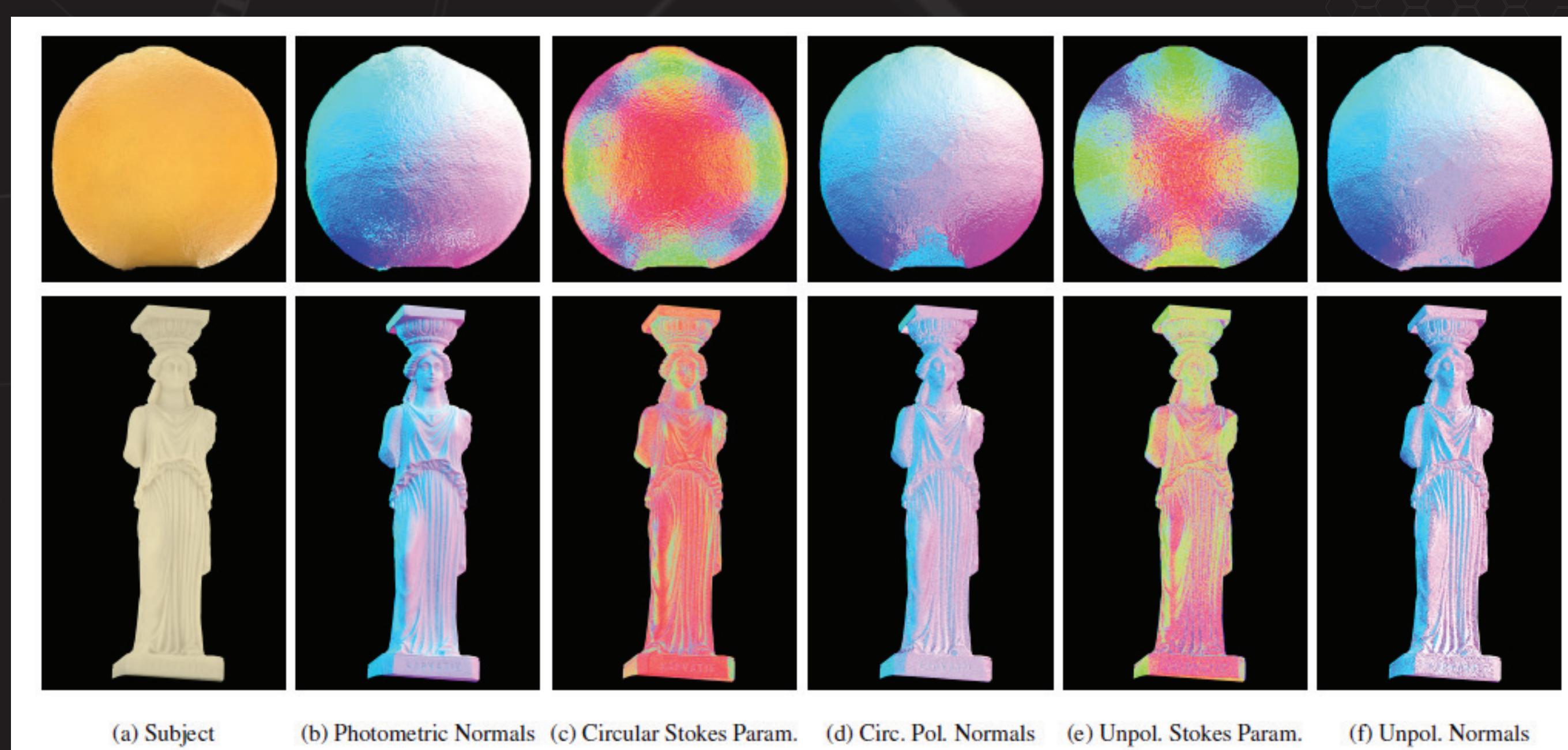


Fig. 1. Estimating surface normals (encoded as  $\frac{1}{2}(\mathbf{x}+1) \rightarrow R$ ,  $\frac{1}{2}(\mathbf{y}+1) \rightarrow G$ , and  $\frac{1}{2}(\mathbf{z}+1) \rightarrow B$ ) from Stokes parameters (encoded as  $|s_3| \rightarrow R$ ,  $|s_1| \rightarrow G$ , and  $|s_2| \rightarrow B$ ) of specularly reflected incident spherical illumination. Surface normals inferred from circularly polarized illumination (c-d), and unpolarized illumination (e-f), compared to surface normals obtained from spherical gradient illumination [1](b). Top-row: Plastic orange -  $\phi$  - ambiguity resolved by growing normals inward. Bottom-row: Marble statue -  $\phi$  - ambiguity resolved using an additional measurement.

In contrast to previous work, we leverage observations of the view-independent symmetric Stokes reflectance field [2] – which encodes the impact of unpolarized, linearly polarized, as well as circularly polarized reflected light – for estimating surface normals under constant incident spherical illumination. We demonstrate that both circularly polarized and unpolarized incident lighting can be used to reliably estimate surface normals from observations of the Stokes reflectance field, and show how this theory can be applied to normal estimation under uncontrolled outdoor illumination.

## Circularly Polarized Incident Lighting

Applying Mueller calculus for specular surface interactions yields the Stokes reflectance vector  $\mathbf{S}' = (s'_0, s'_1, s'_2, s'_3)$ . Under circularly polarized incident light, we can compute the incident angle  $\theta$  of the surface normal by establishing a relationship  $\chi$  between the Stokes components  $s'_1$ ,  $s'_2$  and  $s'_3$ .

$$\chi = \arctan\left(\frac{s'_3}{\sqrt{s'_1{}^2 + s'_2{}^2}}\right) = \arctan\left(\frac{\mp 2\sqrt{R_{\parallel}R_{\perp}}}{R_{\perp} - R_{\parallel}}\right) \quad (1)$$

Note that  $\chi$  is implicitly related by a non-linear one-to-one mapping to  $\theta$  via the Fresnel equations  $R_{\perp}$  and  $R_{\parallel}$ . We invert this non-linear mapping by precomputing a lookup table that maps  $\chi$  to  $\theta$ , obtained by evaluating Eq. (1) for a dense sample of  $\theta$ , and assuming a fixed index of refraction of 1.4.

The corresponding azimuthal angle  $\phi$  of the surface normal can be directly computed from the linear components  $s'_1$ ,  $s'_2$ :

$$\arctan\left(\frac{s'_2}{s'_1}\right) = \arctan\left(\frac{\sin 2\phi}{\cos 2\phi}\right) = 2\phi \quad (2)$$

This relation is ambiguous:  $\phi$  and  $\phi + \pi$  both satisfy the above equation. We employ two alternative strategies for resolving this ambiguity:

1. For convex objects, we can grow the normals in from the silhouette.
2. Alternatively, we can capture an additional photograph of the surface while lit by a spherical gradient illumination condition orthogonal to the viewing direction, such that  $I(\phi, \theta) \neq I(\phi + \pi, \theta)$ .

## Uniform Unpolarized Incident Lighting.

For unpolarized incident lighting,  $\phi$  can be computed similarly as before using Eq. (2). However, Eq. (1) cannot be employed for estimating  $\theta$ , because the circular Stokes component  $s'_3$  differs. While no circular polarization is predicted by Mueller calculus, we experimentally detected a small quantity of left circularly polarized reflectance under unpolarized incident illumination.

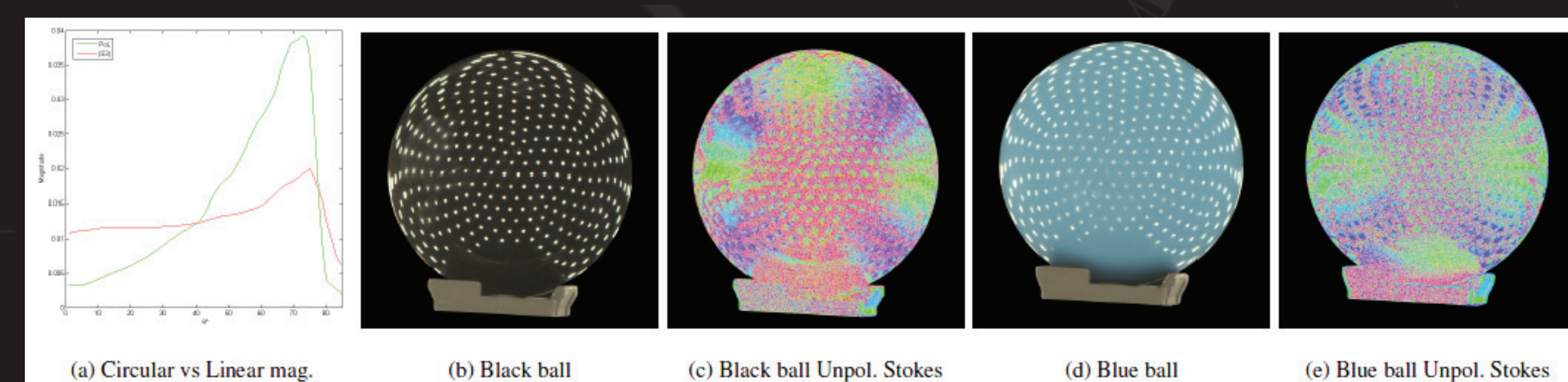


Fig. 2. (a) Plot of reflected circular vs linear polarization under uniform spherical illumination as function of  $\phi$ . (b - e) Stokes parameters of two sharp specular balls under uniform spherical illumination (emitted from a LED sphere with 346 lights) showing circular polarization (red) between the observed specular highlights which are linearly polarized.

We believe that this observed circularly polarized reflectance is due to polarization preserving (subsurface) scattering. Fig. 2, (b-e) shows an experimental validation that indicates that the observed circularly polarized reflectance is not due to specular reflections. Instead of relying on an exact formulation of  $s'_3$  for computing  $\theta$ , an example-based strategy is employed. The Stokes reflectance field of a dielectric object with known shape (e.g., a sphere) is recorded and employed for estimating  $\theta$ .

## Uncontrolled Outdoor Illumination



Fig. 3. Estimated surface normals from Stokes parameters of diffuse outdoor illumination. Toprow: Plastic orange. Bottom-row: Confucius statue.

To extend the proposed method to uncontrolled outdoor environments, we make two observations:

1. We observe that overcast sky is unpolarized, and the content varies approximately as:  $I(\phi, \theta) \sim \sin(\phi)$ . Such an illumination condition is slowly varying, and fulfills  $I(\phi, \theta) \neq I(\phi + \pi, \theta)$ , and thus helps resolve the  $\phi$  ambiguity.
2. Furthermore, if the content of the environment lighting varies slowly in comparison to the sharpness of the specular reflection (which is the case for overcast sky), then we can approximate the intensity of incident lighting over the solid angle of specular response as a constant scale factor  $s_w$ .

Again, we can readily apply the theory outlined for the uniform unpolarized incident lighting case to compute surface normals under uncontrolled overcast illumination. We propose to capture an exemplar in the same environment (and hence it includes  $s_w$ ) for calibration.

Fig. 3 shows results of surface normal estimation from outdoor illumination on a cloudy day for the convex plastic orange as well as a jade Confucius statue with several concavities. The exemplar sphere was captured under the same lighting condition.

## Additional Results

To measure the Stokes parameters, four photographs of a surface are recorded with four different polarizers in front of the camera: a linear polarizer rotated 0° (PH), 45° (P45), 90° (PV), and a (left) circular polarizer (P<sub>c</sub>) [2].

Fig. 4 shows normal estimation results under a spherical linear intensity gradient in the top-down direction, which simulates an idealized outdoor overcast condition.

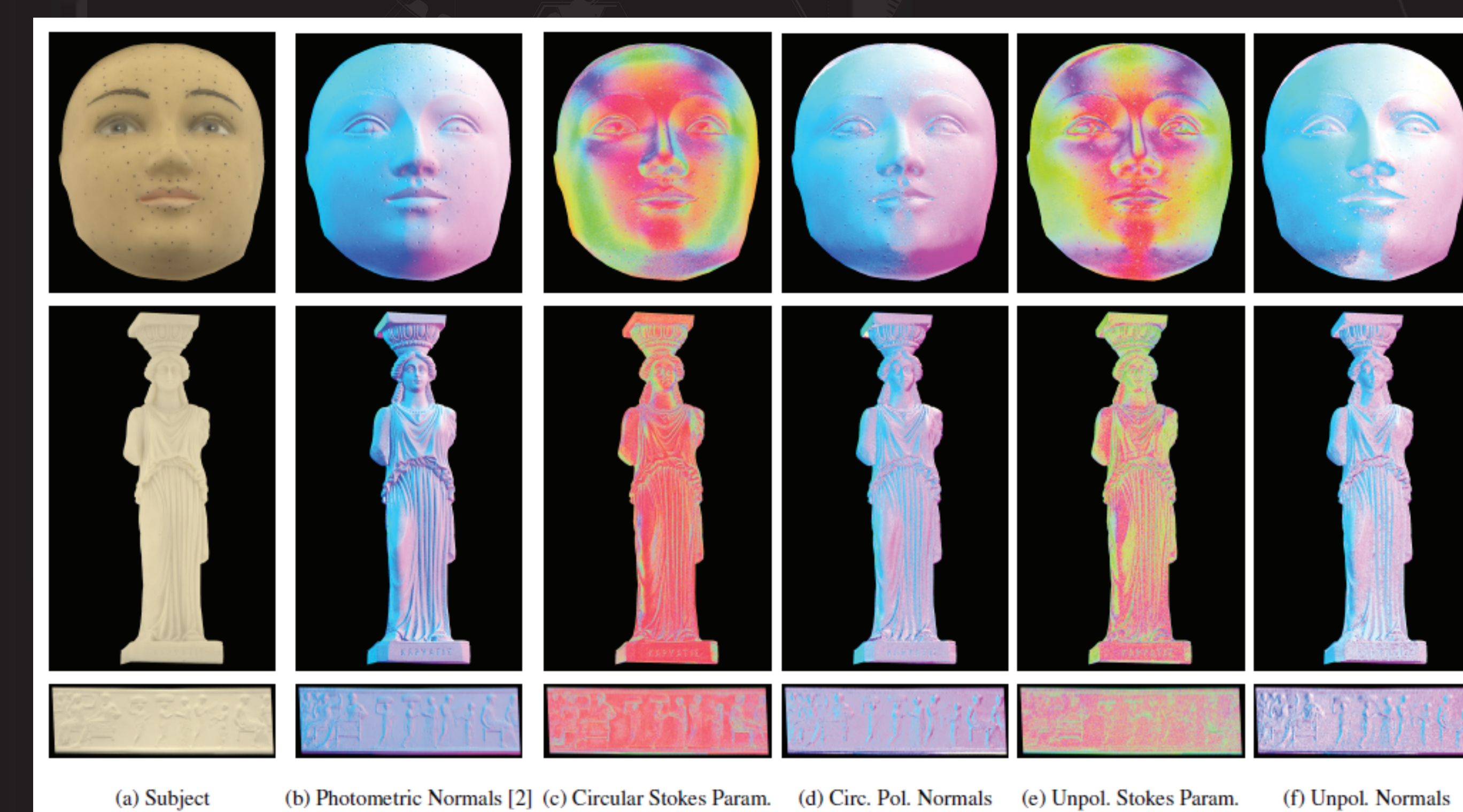


Fig. 4. Estimated surface normals from Stokes parameters under idealized simulated outdoor lighting conditions. Surface normals inferred from circularly polarized illumination (c-d), and unpolarized illumination (e-f), compared to photometric normals [1] (b). Top-row: Plastic maquette - surface normal map of face grown inward from the silhouette. Center-row: Marble statue -  $\phi$  ambiguity resolved using directional cues from the incident illumination. Bottom-row: Plaster bas relief.

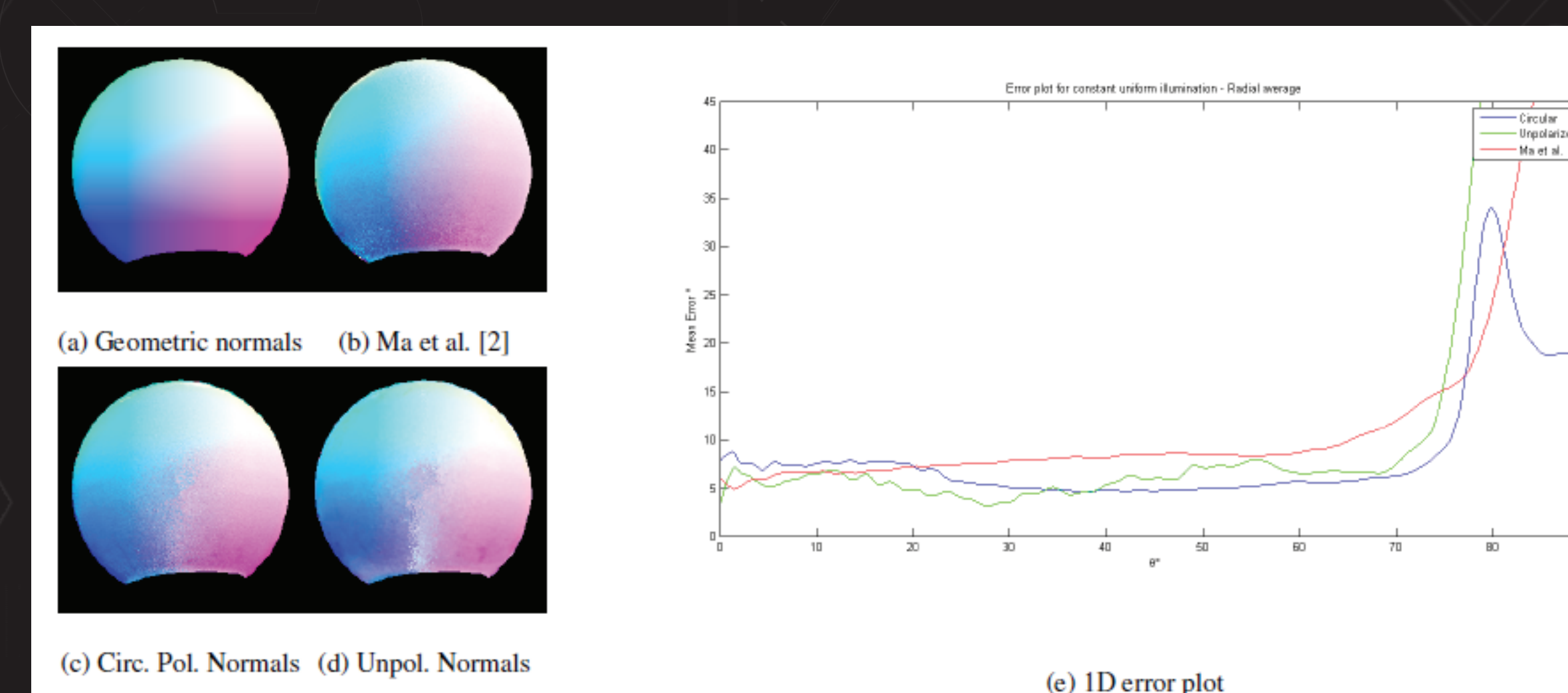


Fig. 5. Surface normals of a spherical ball estimated from Stokes parameters of incident spherical illumination. Surface normals inferred from circularly polarized illumination (c), and unpolarized illumination (d), compared to known ground truth geometric normals (a), and compared to normals obtained from linearly polarized incident lighting using the method of [1] (b).

The inclusion of circular polarization yields a more robust estimation of surface normals compared to prior work which rely solely on linear polarization cues. We found the estimation error to reduce by 5° close to normal incidence with the inclusion of circular polarization.

## References

1. Ma, W.C., Hawkins, T., Peers, P., Chabert, C.F., Weiss, M., Debevec, P.: Rapid acquisition of specular and diffuse normal maps from polarized spherical gradient illumination. In: Rendering Techniques. (2007) 183–194
2. Ghosh, A., Chen, T., Peers, P., Wilson, C.A., Debevec, P.: Circularly polarized spherical illumination reflectometry. ACM Trans. Graph. 29 (2010) 162:1–162:12