Content-Adaptive Image Downscaling
Supplementary Document

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(w_i, h_i)	input (*high* resolution) image dimensions
(w_o, h_o)	output (*low* resolution) image dimensions
(x_k, y_k)	2D kernel indices; $(x_k, y_k) = (k \bmod w_o + \frac{1}{2}, \lfloor k/w_o \rfloor + \frac{1}{2})$
(r_x, r_y)	input/output dimension ratios; $(r_x, r_y) = (w_i/w_o, h_i/h_o)$
k, n	kernel indices; $k, n \in [0, w_o \cdot h_o - 1]$
μ_k	spatial mean
Σ_k	spatial covariance
ν_k	color mean
σ_k	color variance
R_k	set of pixel indices where kernel k can become non-zero; $R_k = \{x + yw_i \mid 0 \le x < w_i, 0 \le y < h_i, \lvert x - x_k \rvert < 2r_x, \lvert y - y_k \rvert < 2r_y\}$
N_k^4	Set of indices of the 4-neighbors of kernel k
N_k^8	Set of indices of the 8-neighbors of kernel k
i	pixel index; $i \in [0, w_i \cdot h_i - 1]$
$\mathbf{p}_i$	pixel location; $p_i = (i \bmod w_i, \lfloor i/w_i \rfloor)$
$\mathbf{c}_i$	CIELAB color of pixel i
$w_k(i)$	Value of kernel k at pixel i
$\gamma_k(i)$	Value of *normalized* kernel k at pixel i

Figure 1: *Symbols used throughout the paper and in the pseudocode*

1 Pseudocode

This document provides commented pseudocode for the algorithm described in the main paper, published in SIGGRAPH Asia 2013. Refer to Figure 1 for a list and explanations of symbols. The code here is complete and efficient, e.g. kernel computations are clamped to ranges where they take on non-zero values. However, for maximum clarity we did not always combine all loops wherever possible.

The function clamp$(x, minVal, maxVal)$ constrains the scalar x to lie within the range specified. Similarly, the function clampBox$(\mathbf{v}, Box)$ constrains the 2-vector $\mathbf{v}$ to lie within Box. The function SVD(Σ) performs singular value decomposition.

// The main program
procedure DOWNSCALING
    INITIALIZE
    loop
        E-STEP
        M-STEP
        C-STEP
        if no changes in last M-Step or C-Step **then return**
    end loop
end procedure

// Initialize all variables
procedure INITIALIZE
    for all k **do**
        $\mu_k \leftarrow (x_k, y_k)^\top$
        $\Sigma_k \leftarrow \begin{bmatrix} r_x/3 & 0 \\ 0 & r_y/3 \end{bmatrix}$
        $\nu_k \leftarrow (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})^\top$
        $\sigma_k \leftarrow 10^{-4}$
    end for
end procedure

// Expectation step
procedure E-STEP
    // Compute all kernels
    for all k **do**
        for all $i \in R_k$ **do**
            $w_k(i) \leftarrow \exp\left(-\frac{1}{2}(\mathbf{p}_i - \mu_k)^\top \Sigma_k^{-1}(\mathbf{p}_i - \mu_k) - \frac{\lVert \mathbf{c}_i - \nu_k \rVert^2}{2\sigma_k^2}\right)$
        end for
        $w_{sum} \leftarrow \sum_{i \in R_k} w_k(i)$
        for all $i \in R_k$ **do**
            $w_k(i) \leftarrow w_k(i)/w_{sum}$
        end for
    end for
    // Normalize per pixel
    for all i **do**
        for all k **with** $i \in R_k$ **do**
            $\gamma_k(i) \leftarrow w_k(i)/\sum_n w_n(i)$
        end for
    end for
end procedure

// Maximization step
procedure M-STEP
for all k **do**
$w_{sum} \leftarrow \sum_i \gamma_k(i)$
$\Sigma_k \leftarrow \frac{1}{w_{sum}} \sum_i \gamma_k(i)(\mathbf{p}_i - \boldsymbol{\mu}_k)(\mathbf{p}_i - \boldsymbol{\mu}_k)^\top$
$\boldsymbol{\mu}_k \leftarrow \frac{1}{w_{sum}} \sum_i \gamma_k(i)\mathbf{p}_i$
$\boldsymbol{\nu}_k \leftarrow \frac{1}{w_{sum}} \sum_i \gamma_k(i)\mathbf{c}_i$
end for
end procedure

// Correction step
procedure C-STEP
// Spatial constraints
for all k **do**
$\overline{\boldsymbol{\mu}^k} \leftarrow \frac{\sum_{n \in N_k^4} \boldsymbol{\mu}_n}{|N_k^4|}$
end for
for all k **do**
$\boldsymbol{\mu}_k \leftarrow \text{clampBox}\left(\frac{1}{2}\boldsymbol{\mu}_k + \frac{1}{2}\overline{\boldsymbol{\mu}_k},\ (x_k, y_k)^\top \pm (\frac{r_x}{4}, \frac{r_y}{4})^\top\right)$
end for
// Constrain spatial variance
for all k **do**
$(U, S, V^\star) \leftarrow \text{SVD}(\Sigma_k)$
$S_{1,1} \leftarrow \text{clamp}\left(S_{1,1},\ 0.05,\ 0.1\right)$
$S_{2,2} \leftarrow \text{clamp}\left(S_{2,2},\ 0.05,\ 0.1\right)$
$\Sigma_k \leftarrow USV^\star$
end for
// Shape constraints
for all k **do**
for all $n \in N_k^8$ **do**
$\mathbf{d} \leftarrow (x_n - x_k, y_n - y_k)^\top$
// Directional variance
$s \leftarrow \sum_{i \in R_k} \gamma_k(i) \max\left(0,\ (\mathbf{p}_i - \boldsymbol{\mu}_k)^\top \mathbf{d}\right)^2$
// Edge strength
$f \leftarrow \sum_{i \in R_k} \gamma_k(i)\gamma_n(i)$
// Edge orientation
$\mathbf{o} \leftarrow \sum_{i \in R_k} \nabla \frac{\gamma_k(i)}{\gamma_k(i) + \gamma_n(i)}$
// Check for dominant kernels and staircasing
if $s > 0.2 r_x$ **or** $(f < 0.08$ **and** $\angle(\mathbf{d}, \mathbf{o}) > 25°)$ **then**
$\sigma_k \leftarrow 1.1\sigma_k$
$\sigma_n \leftarrow 1.1\sigma_n$
end if
end for
end for
end procedure