Interarrival Times Characterization and Fitting for Markovian Traffic Analysis

Giuliano Casale[1], Eddy Zheng Zhang[1], Evgenia Smirni[1]

College of William and Mary, Computer Science Dept.
23187-8795 Williamsburg, VA, US
casale,eddy,esmirni@cs.wm.edu

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1 Introduction

Markovian models provide a convenient way of evaluating the performance of network traffic since their queueing analysis enjoys established theoretical results and efficient solution algorithms [1]. Although unable to directly generate traffic with long-range dependent (LRD) behavior, Markovian models can approximate accurately LRD traffic in several ways, e.g., by superposition of flows with short-range dependent (SRD) behavior over many time scales. This is known to be sufficient for the evaluation of real systems since the performance effects of LRD traffic becomes nil beyond a finite number of time scales [2].

One of the main obstacles to the Markovian analysis of network traffic is model parameterization, which often involves describing the interaction of several tens or hundreds of states. Even for basic Markov Modulated Poisson processes (MMPP) or phase-type renewal processes (PH), few results exist for exact parameterization and they are restricted to models of two or three states [3–7]. Due to the lack of characterization results for larger-state-space models, some earlier works have focused on fitting network traffic models by parameterizing Markovian Arrival Processes (MAPs) or MMPPs with exactly two or three states [4–8]. The small state space minimizes the costs of queueing analysis, but places significant assumptions on the form of the autocorrelations.

In [9] Andersen and Nielsen develop a general fitting algorithm to model LRD traffic traces by superposition of several MMPP(2) sources [10]. The algorithm has low computational costs but only matches first and second order descriptors of the counting process. Following a different approach, Horváth and Telek [11] consider the multifractal traffic model of Riedi et al. [12], and obtain a class of MMPPs which exhibits multifractal behavior [13]. Simulation results on the Bellcore Aug89 trace show that this algorithm achieves better accuracy than the superposition method in [9], but at the expense of a larger state space.

To tackle the above issues, we develop new characterization and fitting methods for MAPs. We first characterize the general properties of interarrival time (IAT) processes using a spectral approach. Based on this characterization, we show how different MAP processes can be combined together using Kronecker products to define a larger MAP with predefined properties of interarrival times.

We then devise an algorithm based on this Kronecker composition method, which can be customized to fit an arbitrary number of moments and to meet the desired cost-accuracy tradeoff. Numerical results of the fitting algorithm on real HTTP and TCP traffic data, such as the Bellcore Aug89 trace, indicate that the proposed fitting methods achieve increased prediction accuracy with respect to other state-of-the-art fitting methods.

2 IAT Processes in MAPs

2.1 Definitions

A MAP(n) is specified by two $n \times n$ matrices: a stable matrix $\boldsymbol{D}_0$ and a nonnegative matrix $\boldsymbol{D}_1$ that describe transition rates between n states. Each transition in $\boldsymbol{D}_1$ produces a job arrival; $\boldsymbol{D}_0$ describes instead background transitions not associated with arrivals. The matrix $\boldsymbol{Q} = \boldsymbol{D}_0 + \boldsymbol{D}_1$ is the infinitesimal generator of the underlying Markov process.

We focus on the interval stationary process that describes the Inter-Arrival Times(IATs). For a MAP(n), this is described by the embedded discrete-time chain with irreducible stochastic matrix $\boldsymbol{P} = (-\boldsymbol{D}_0)^{-1}\boldsymbol{D}_1$, with probability vector $\boldsymbol{\pi}_e$, where $\boldsymbol{\pi}_e \boldsymbol{e} = 1$ and $\boldsymbol{e}$ is a column vector of 1's of the appropriate dimension. Then, its IAT is phase-type distributed with k-th moment

$$E[X^k] = k!\boldsymbol{\pi}_e(-\boldsymbol{D}_0)^{-k}\boldsymbol{e}, \quad k \geq 0. \tag{1}$$

The lag-k autocorrelation coefficient is

$$\rho_k = (E[X]^{-2}\boldsymbol{\pi}_e(-\boldsymbol{D}_0)^{-1}\boldsymbol{P}^k(-\boldsymbol{D}_0)^{-1}\boldsymbol{e} - 1)/\mathrm{CV}^2. \tag{2}$$

Higher order moments of the IAT process can also be described as special cases of *joint moments*. Let X_i be the i-th IAT from an arbitrary starting epoch $i_0 = 0$, and consider a sequence $X_{i_1}, X_{i_2}, \ldots, X_{i_L}$, where $0 \leq i_1 < i_2 < \ldots < i_L$. The moments of L consecutive IATs are given by

$$H(\boldsymbol{i}, \boldsymbol{k}) = E[X_{i_1}^{k_1} X_{i_2}^{k_2} \cdots X_{i_L}^{k_L}],$$

where $\boldsymbol{i} = (i_1, i_2, \ldots, i_L)$ and $\boldsymbol{k} = (k_1, k_2, \ldots, k_L)$. The moments $H(\boldsymbol{i}, \boldsymbol{k})$ capture nonlinear temporal relations between samples and are known to completely characterize a MAP [14, 15].

2.2 Spectral Characterization of Moments

We obtain a spectral representation of moments for MAPs, a simple scalar representation of (1) based on spectral properties of $(-\boldsymbol{D}_0)^{-1}$. This allows to represent the MAPmoments in terms of few scalar parameters, rather than by formulas using matrices. We begin by describing the moments (1) in terms of the spectrum of $(-\boldsymbol{D}_0)^{-1}$. Recall that the characteristic polynomial of a $n \times n$ matrix $\boldsymbol{A}$ is

$$\phi(\boldsymbol{A}) = s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1}s + \alpha_n,$$

which is a polynomial in s with roots s_i equal to the eigenvalues of $\mathbf{A}$. Consider the Cayley-Hamilton theorem [16], by which the powers of $\boldsymbol{A}$ satisfy $\boldsymbol{A}^k = -\sum_{j=1\ldots n} \alpha_j \boldsymbol{A}^{k-j}$ for $k \geq n$ that is, matrix powers are linearly dependent. Because MAP moments are computed in (1) from powers of $(-\boldsymbol{D}_0)^{-1}$, they are linearly dependent.

Lemma 1. *In a MAP(n), any $n+1$ consecutive moments are linearly dependent according to the relation*

$$E[X^k] = -\sum_{j=1\ldots n} b_j E[X^{k-j}],\ E[X^0] = 1,\ k \geq n, \tag{3}$$

where $b_j = m_j k!/(k-j)!$, and m_j is the coefficient of s^{n-j} in $\phi((-\boldsymbol{D}_0)^{-1})$.

Observing that (3) is an homogeneous linear recurrence of order n in $E[X^k]/k!$ with constant coefficients m_j, and m_j are functions of the eigenvalues of $(-\boldsymbol{D}_0)^{-1}$, we can derive a closed-form formula for $E[X^k]$ (see proof in [17]).

Theorem 1. *Let $(-\boldsymbol{D}_0)^{-1}$ have $m \leq n$ distinct eigenvalues $\theta_t \in \mathbb{C}$, $1 \leq t \leq m$. Let q_t be the algebraic multiplicity of θ_t, $\sum_{t=1\ldots m} q_t = n$. Then the IAT moments are given by*

$$E[X^k] = \sum_{t=1\ldots m} k!\, \theta_t^k \sum_{j=1\ldots q_t} M_{t,j} k^{j-1}, \tag{4}$$

$$E[X^0] = \sum_{t=1\ldots m} M_{t,1} = 1, \tag{5}$$

where the constants $M_{t,j}$'s are independent of k. In particular,

$$M_{t,1} = \boldsymbol{\pi}_e (-\boldsymbol{D}_0)_t^{-1} \boldsymbol{e}, \tag{6}$$

where $(-\boldsymbol{D}_0)_t^{-1}$ is the t-th spectral projector of $(-\boldsymbol{D}_0)^{-1}$, i.e., the product of the right and left eigenvectors for θ_t.

2.3 Spectral Characterization of Autorrelations

The spectral characterization can be extended to autocorrelations using the properties of the powers $\boldsymbol{P}^k$ in (2). Analogous to the procedures obtaining spectral characterization of moments, we first establish a linear recurrence formula for $n+1$ consecutive autocorrelations and then derive closed form formulas for ρ_k.

Theorem 2. *Let $\gamma_t \in \mathbb{C}$, $1 \leq t \leq m$, be an eigenvalue of $\boldsymbol{P}$ with algebraic multiplicity r_t. If $\gamma_t = 0$ assume that its geometric multiplicity equals its algebraic multiplicity, i.e., the r_t associated Jordan blocks have all order one. Then the autocorrelation function of a MAP is*

$$\rho_k = \sum_{t=2\ldots m} \gamma_t^k \sum_{j=1\ldots r_t} A_{t,j} k^{j-1}, \quad k \geq 1 \tag{7}$$

$$\rho_0 = \sum_{t=2\ldots m} A_{t,1} = (1 - 1/\mathrm{CV}^2)/2, \tag{8}$$

where the $A_{t,j}$'s constants are independent of k. In particular,

$$A_{t,1} = E[X]^{-2}\boldsymbol{\pi}_e(-\boldsymbol{D}_0)^{-1}\boldsymbol{P}_t(-\boldsymbol{D}_0)^{-1}\boldsymbol{e}/\mathrm{CV}^2, \tag{9}$$

in which $\boldsymbol{P}_t$ is the t-th spectral projector of $\boldsymbol{P}$, that is, the product of the right and left eigenvectors associated to γ_t.

2.4 Compositional Properties of Moments and Autocorrelations

We define a new composition method for combining two MAPs into one larger MAP. Using this method, the moments and autocorrelations of the newly composed MAP can be readily computed from formulas involving only the moments and autocorrelations of the composition MAPs. This essentially facilitates the composition of small processes for the purpose of predefined moments and autocorrelations in all orders.

The new composition method is based on the Kronecker product of matrices and it applies to the operation of MAPs, we call it the Kronecker product of MAPs. Let $\mathrm{MAP}_a = \{\boldsymbol{D}_0^a, \boldsymbol{D}_1^a\}$ and $\mathrm{MAP}_b = \{\boldsymbol{D}_0^b, \boldsymbol{D}_1^b\}$ be MAPs of order n_a and n_b, respectively, and assume at least one of the two MAPs has a diagonalized $\boldsymbol{D}_0$, without loss of generality, assume $\boldsymbol{D}_0^b$ is a diagonal matrix. The Kronecker Product of MAP_a and MAP_b is defined as

$$\mathrm{MAP}_a \otimes \mathrm{MAP}_b = \{-\boldsymbol{D}_0^a \otimes \boldsymbol{D}_0^b, \boldsymbol{D}_1^a \otimes \boldsymbol{D}_1^b\},$$

which has been proved to be a valid MAP of order $n_a n_b$ in [17]. Theorem 3 provides formulas relating the statistics of the composed process and composition processes, provable via the basic eigenvalue properties of Kronecker product and the spectral characterization of MAPs.

Theorem 3. *Moments and autocorrelations of $MAP_a \otimes MAP_b$, where at least one of the MAP_a and MAP_b has diagonalized D_0, satisfy*

$$E[X^k] = E[X_a^k]E[X_b^k]/k!, \tag{10}$$

$$CV^2\rho_k = (CV_a^2)\rho_k^a + (CV_b^2)\rho_k^b + (CV_a^2 CV_b^2)\rho_k^a\rho_k^b, \tag{11}$$

where the quantities in the right-hand side refer to MAP^a and MAP^b. In particular the relation for $E[X^k]$ immediately implies

$$1 + CV^2 = (1 + CV_a^2)(1 + CV_b^2)/2. \tag{12}$$

The relationship between moments of the composed MAP process and the composition MAP processes generalizes in the similar fashion to the joint moments.

Theorem 4. *The joint moments of $MAP_a \otimes MAP_b$,where at least one of the MAP_a and MAP_b has diagonalized D_0, satisfy*

$$H(\boldsymbol{i}, \boldsymbol{k}) = \frac{H^a(\boldsymbol{i}, \boldsymbol{k})H^b(\boldsymbol{i}, \boldsymbol{k})}{k_1!k_2!\cdots k_L!}, \tag{13}$$

being $H^a(\boldsymbol{i}, \boldsymbol{k})$ and $H^a(\boldsymbol{i}, \boldsymbol{k})$ the joint moments of $\{\boldsymbol{D}_0^a, \boldsymbol{D}_1^a\}$ and $\{\boldsymbol{D}_0^b, \boldsymbol{D}_1^b\}$, respectively.

3 Application of IAT Properties in MAP Process Design

Based on the compositional IAT properties, we propose a general process composition method called Kronecker Product Composition (KPC), which aims at integrating more than two MAPs into one large MAP. Given J MAPs $\{\boldsymbol{D}_0^j, \boldsymbol{D}_1^j\}$, we define the KPC process as the MAP

$$\{\boldsymbol{D}_0^{\mathrm{kpc}}, \boldsymbol{D}_1^{\mathrm{kpc}}\} = \{(-1)^{J-1}\boldsymbol{D}_0^1 \otimes \cdots \otimes \boldsymbol{D}_0^J, \boldsymbol{D}_1^1 \otimes \cdots \otimes \boldsymbol{D}_1^J\}.$$

To generate a valid KPC process, we require at least $J-1$ composing processes have diagonalized $\boldsymbol{D}_0^j$ according to Theorem 3. Nevertheless, because one MAP can be arbitrary, the KPC does not place modeling restrictions.

Using KPC, we define a general-purpose fitting algorithm for network traffic. We illustrate the algorithm in the case where the J composing MAPs used in the KPC are an arbitrary MAP(2) and $J-1$ MAP(2)s with diagonal $\boldsymbol{D}_0$, but the method works with minor modifications also with other processes. The algorithm proceeds in three steps: (1) Step 1-We fit autocorrelations and CV^2 of J MAPs by a least sqaure optimization algorithm constrained to the properties of the KPC; (2) Step 2-Given fixed autocorrelation and CV^2 there exist many possible valid processes; we thus solve a new nonlinear optimization program to select $E[X]$ and $E[X^3]$ that results in better fitting of higher order properties of IATs on a set of sample joint moments; (3) Step 3-Given the target optimal values for the $E[X](j), \mathrm{CV}^2(j), E[X^3](j), \gamma_2(j)$ we generate the J feasible MAPs and compute the final process by KPC.

We show the effectiveness of our KPC fitting algorithm using the Bellcore Aug89 trace on a first-come-first-served queue with deterministic service and different utilization levels. This is the standard case for evaluating the quality of LRD trace fitting, e.g., [9, 11, 18]. The traffic trace consists of 10^6 IAT samples collected in 1989 at the Bellcore Morristown Research and Engineering facility and shows a clear LRD behavior [19]. We present a comparison of our algorithm with the best-available algorithms for Markovian analysis of LRD traffic, that is, the method of Andersen and Nielsen (A&N) in [9] and the multifractal approach of Horvath and Telek (H&T) in [11].

We run the KPC fitting program described above and determine a MAP(16) which accurately fits the trace. The size of this MAP is similar to those employed in previous work, which are composed by 16 states (A&N) or 32 states (H&T). The values of the first three moments of the MAP(16) are given in Table 1. We compare the queueing prediction of the three models for utilization levels of 20%, 50%, and 80%. In Figure 1 we plot the complementary cdf (ccdf) of queue-length probabilities Pr(queue $\geq x$), which accounts also for the residual queueing probability mass and thus shows the impact of the tail probability.

At 20% utilization, our method gives almost the same results of the multifractal technique, while the method of A&N seems to underestimate the queueing probability for the smallest values of x. The intermediate case for 50% utilization is generally difficult to capture, since the network is approaching heavy traffic, but the dependence effects are still not as strong as in slightly higher utilization

Table 1. MAP(16) fitting of the Bellcore Aug89 Trace using the KPC algorithm

BC-Aug89	Trace	MAP(16)
$E[X]$	$3.1428 \cdot 10^{-3}$	$3.1428 \cdot 10^{-3}$
CV^2	$3.2236 \cdot 10^{0}$	$3.2235 \cdot 10^{0}$
$E[X^3]$	$2.0104 \cdot 10^{-6}$	$1.1763 \cdot 10^{-5}$
γ_2	n/a	$9.9995 \cdot 10^{-1}$

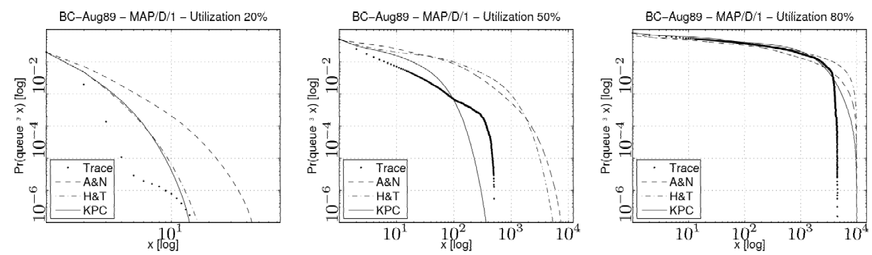

Fig. 1. Queueing predictions for the Bellcore Aug89 trace on a queue with deterministic service.

values, i.e., for 60% – 70% utilization (see, e.g., [9]). All methods initially overestimate the real probability, but for higher values of x our method is closer to the trace values than A&N and H&T which predict a large probability mass also after $x = 10^3$. In the case of 80% utilization all three methods perform well, with our algorithm and the H&T being more precise than A&N. The final decay of the curve is again similar, but the KPC method resembles better the simulated trace.

Overall, the result of this trace indicates that the KPC approach seems more effective than both H&T and the A&N methods, while preserving the smallest representation (16 states) of the A&N method. It also interesting to point out that the fitting leaves room for further improvements, especially in the 50% case which is difficult to approximate. This may indicate that significant information about the IAT process may be captured by statistics of higher order than the bispectrum.

4 Conclusion

We have presented several contributions to the Markovian traffic analysis. We have obtained a spectral characterization of moments and autocorrelation which simplifies the analysis of MAP processes. In the second part of the paper, we have studied the definition of large MAPs by Kronecker Product Composition (KPC), and shown that this provides a simple way to create processes with predefined moments and correlations at all orders. Detailed comparisons with other state-of-the-art fitting methods show that KPC provides improved fitting of LRD trace.

References

1. Nelson, R.D.: Probability, Stochastic Processes and Queueing Theory. Springer-Verlag (1995)
2. Grossglauser, M., Bolot, J.: On the relevance of long-range dependence in network traffic. In: SIGCOMM. (1996) 15–24
3. Berger, A.W.: On the index of dispersion for counts for user demand modeling. In: ITU, Madrid, Spain, AT&T Study Group 2, Question 17/2. (1994) 27–29
4. Ferng, H., Chang, J.: Connection-wise end-to-end performance analysis of queueing networks with mmpp inputs. Perform. Eval. **43** (2001) 39–62
5. Kang, S., Yong, H., Sung, D., Choi, B.: An application of markovian arrival process (map) to modeling superposed atm cell streams. IEEE Transactions on Communications **50** (2002) 633–642
6. Heindl, A.: Analytic moment and correlation matching for map(2)s. In: Proc. 6th Int. Workshop on Performability Modeling of Computer and Communication Systems (PMCCS). (2003) 39–42
7. Heindl, A., Mitchell, K., van de Liefvoort, A.: Correlation bounds for second-order maps with application to queueing network decomposition. Perform. Eval. **63** (2006) 553–577
8. Diamond, J., Alfa, A.: On approximating higher-order MAPs with MAPs of order two. Queueing Systems **34** (2000) 269–288
9. Andersen, A.T., Nielsen, B.F.: A markovian approach for modeling packet traffic with long-range dependence. IEEE JSAC **16** (1998) 719–732
10. Heffes, H., Lucantoni, D.M.: A markov modulated characterization of packetized voice and data traffic and related statistical multiplexer performance. IEEE JSAC **4** (1986) 856–868
11. Horváth, A., Telek, M.: Markovian modeling of real data traffic: Heuristic phase type and map fitting of heavy tailed and fractal like samples. In: Performance. (2002) 405–434
12. Riedi, R.H., Crouse, M.S., Ribeiro, V.J., Baraniuk, R.G.: A multifractal wavelet model with application to network traffic. IEEE Transactions on Information Theory **45** (1999) 992–1018
13. Taqqu, M., Teverovsky, V., Willinger, W.: Is network traffic self-similar or multifractal. Fractals **63** (1997) 833–846
14. Andersen, A.T., Nielsen, B.F.: On the use of second-order descriptors to predict queueing behavior of maps. Naval Res. Logistics **49** (2002) 391–409
15. Telek, M., Horváth, G.: A minimal representation of markov arrival processes and a moments matching method. In: Performance. (2007) 1153–1168
16. Hohn, F.E.: Elementary Linear Algebra. Dover (1973)
17. Casale, G., Zhang, E.Z., Smirni, E.: Interarrival times characterization and fitting for markovian traffic analysis. Submitted to IEEE/ACM Transactions on Networking (2007)
18. Horváth, G., Buchholz, P., Telek, M.: A map fitting approach with independent approximation of the inter-arrival time distribution and the lag correlation. In: QEST. (2005) 124–133
19. Leland, W.E., Taqqu, M.S., Willinger, W., Wilson, D.V.: On the self-similar nature of ethernet traffic (extended version). IEEE/ACM Trans. Netw. **2** (1994) 1–15