CSci 243 Homework 1

Your name

Today's date

PROBLEM 1. Give the truth table for $\phi = (\neg p \vee q \vee r) \wedge (p \vee q \vee \neg r)$.

SOLUTION:

p	q	r	$\neg p \vee q \vee r$	$p \vee q \vee \neg r$	ϕ
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

PROBLEM 2. Prove that any nonempty tree has one more nodes than it has edges.

PROOF. We prove by structual induction that for a nonempty tree T with n nodes and e edges, $n = e + 1$.

Basis step: The basis case is $n = 1$. Obviously, a tree with only one node does not have any edges, suggesting $e = 0$. So $n = e + 1$.

Inductive step: We assume that for any tree with less than n nodes the equality holds true. Now onsider a tree T with n nodes. Assume that T contains k subtrees, $T_1, T_2, \ldots, T_k$ and that subtree T_i, for $i = 1, 2, \ldots, k$, has n_i nodes and e_i edges. Notice that $e = \sum_{i=1}^{k} e_i + k$. Since $n_i < n$, then by the induction hypothesis $n_i = e_i + 1$ for all i's. Therefore,

$$\begin{aligned} n &= \sum_{i=1}^{k} n_i + 1 \\ &= \sum_{i=1}^{k} (e_i + 1) + 1 \\ &= \sum_{i=1}^{k} e_i + k + 1 \\ &= e + 1. \end{aligned}$$

This completes the induction.

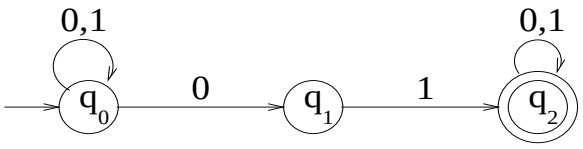

Figure 1: An NFA that accepts all strings with substring 01.

PROBLEM 3. Show an example of how to include a figure in LaTeX.

EXAMPLE: Figure 1 is a state diagram for an NFA, a model of computation that you will learn in CSci 423 Finite Automata.