Simulating Forest Fires

Consider a forest fire model, specified as follows.

- The landscape is an $X \times Y$ grid, with periodic boundary conditions (a doughnut).
- Time (in years) is discrete so that $t = 0, 1, 2, \ldots$
- At any time each (x, y) cell is in one of two states, *clear* (no trees) or *forested* (filled with trees).
- The landscape evolves in time in accordance with two probability parameters and one cluster rule.

  Tree Formation — For each year and for each clear cell, $p < 1$ is the probability that this cell will become forested.

  Lightning Strike — For each year and for each forested cell, $q < p$ is the probability that this cell will be struck by lightning, burn and thereby become clear.

  Cluster Rule — When a forested cell is struck by lightning, the burning in this cell will cause *all* of the forested cells in its *nearest neighbor forested cluster* to also burn and become clear (in parallel, immediately).

Growing The Landscape

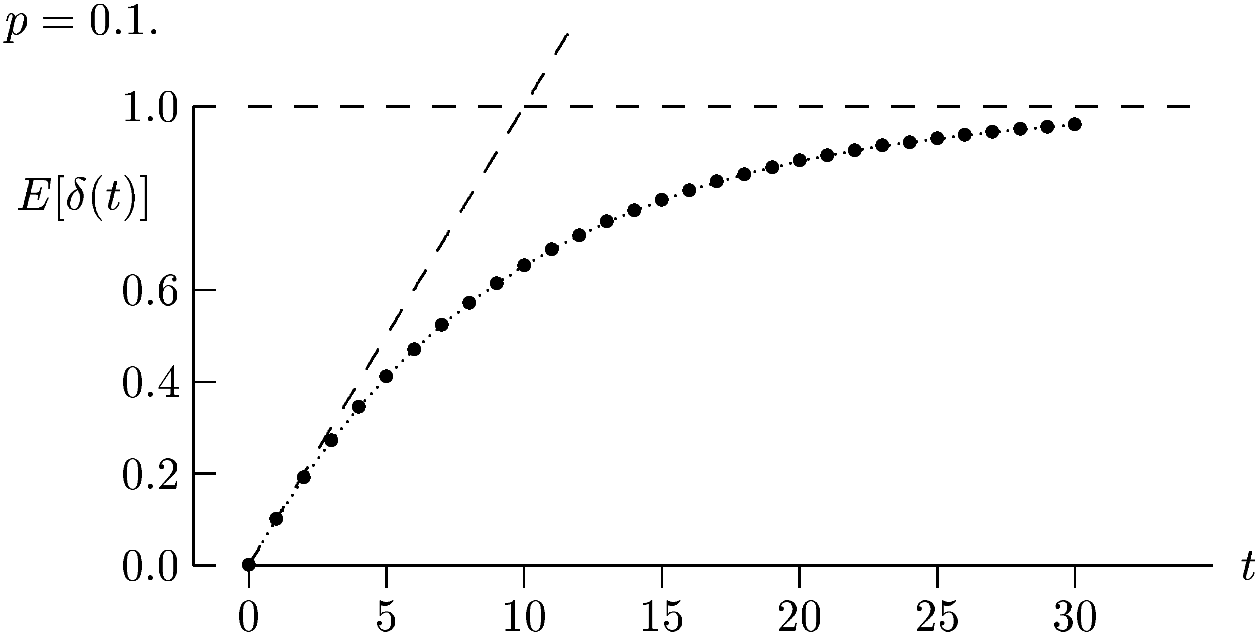
The *proportion* of forested cells at time t is

$$\delta(t) = \frac{\text{the number of forested cells at time } t}{XY}$$

If there is no lightning ($q = 0$) and if the landscape is initially clear (at $t = 0$) then the expected (average) proportion of forested cells at time t is

$$E[\delta(t)] = 1 - (1 - p)^t \qquad t = 0, 1, 2, \ldots$$

So, *in the absence of lightning*, the expected proportion of forested cells will grow asymptotically to 1.0, as illustrated for $p = 0.1$.

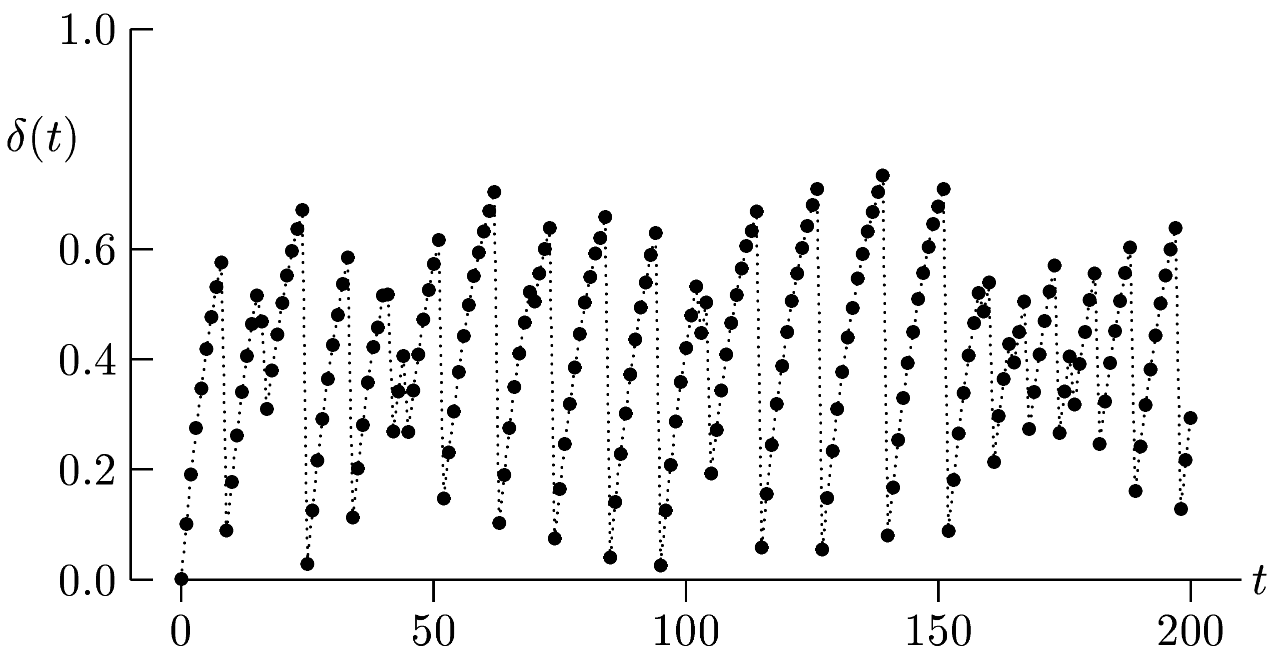


4–6–98, SKP–2

When Lightning Strikes

Provided $q > 0$, lightning will occasionally strike and cause a cluster of forested cells to become clear (burn).

This figure illustrates the simulated 200-year time history of a landscape with $X = Y = 100$, $p = 0.1$ and $q = 1/XY$. Although the time history is stochastic, there is a clear periodic pattern of sustained multi-year tree growth followed by an occasional, and usually catastrophic, lightning strike that clears much of the landscape.



Data Structure

A natural way to represent the landscape is as an $X \times Y$ array

```
int cell[X][Y];
```

with the understanding that `cell[x][y]` equal to 0 represents a *clear* cell, and any other value represents a *forested* cell. In this way the landscape is naturally initialized with all cells clear.

Consistent with this representation, at any time the number of forested cells in the landscape (`trees`) is given by

```
trees = 0;
for (x = 0; x < X; x++)
  for (y = 0; y < Y; y++)
    if (cell[x][y])
      trees++;
```

Because the landscape boundary conditions are periodic, all cells (even those on the boundary) have exactly *four* nearest neighbors, to the North, South, East and West.

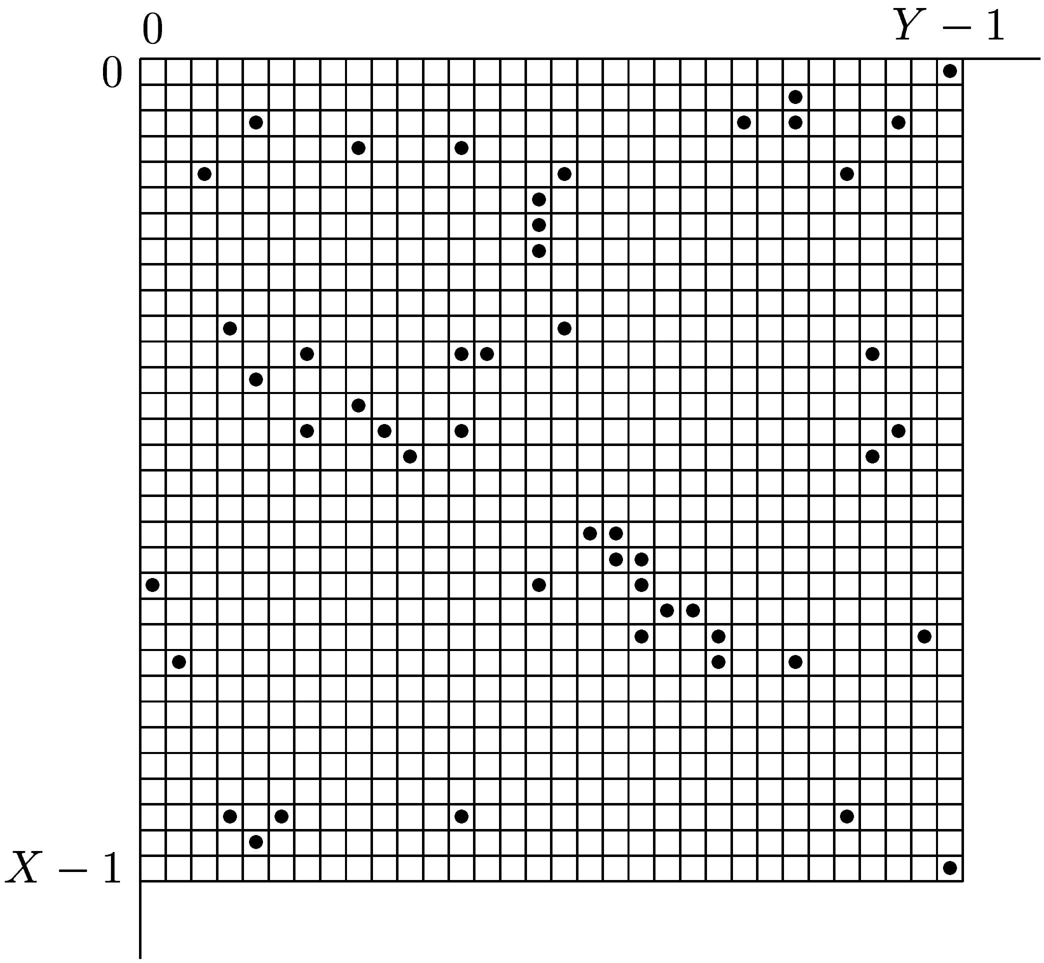
Computational Model

Except for implementing the cluster rule, at the computational level the following version of the forest fire model is easily created and understood.

```
t = 0;
while (t < T) {
  for (x = 0; x < X; x++)
    for (y = 0; y < Y; y++) {
      u = Random();
      if (cell[x][y] == 0) {
        if (u < p) {
          cell[x][y] = 1;
          /* update the associated cluster */
        }
      }
      else if (u < q) {
        cell[x][y] = 0;
        /* clear the associated cluster */
      }
    }
  t++;
}
```

Landscape

Starting with an initially clear landscape, after a few time steps a typical landscape may look like the following. (The •'s indicate forested cells.)



Clusters

Definition — two forested cells are in the same *nearest neighbor forested cluster* ("cluster") if and only if they are connected by a nearest neighbor sequence of forested cells.

¿From this definition, it follows that:

- At any time each forested cell is in exactly one cluster. That is, the clusters form a *partition* of the set of forested cells.
- Equivalently, cluster membership is an *equivalence relation* in the set of forested cells.

If $\mathcal{F}$ represents the set of forested cells, N is the number of clusters, and $\mathcal{F}_0, \mathcal{F}_1, \ldots, \mathcal{F}_{N-1}$ represent the clusters, then each cluster contains at least one cell ($\mathcal{F}_n \neq \emptyset$ for all n), the clusters are *disjoint* ($\mathcal{F}_n \cap \mathcal{F}_{n'} = \emptyset$ for all $n \neq n'$) and

$$\mathcal{F} = \mathcal{F}_0 \cup \mathcal{F}_1 \cup \cdots \cup \mathcal{F}_{N-1}.$$

Dynamic Clusters

The clusters are *dynamic* — the number and structure of the clusters varies with time.

Each time a cell changes from clear to forested:

- either two or more clusters collapse into one because they become nearest neighbor connected by the newly forested cell (and so N decreases by at least one);
- or the size of one cluster increases by one and all the other clusters are unchanged (and so N doesn't change);
- or a new cluster of size one is formed (and so N increases by one).

Each time a cell changes from forested to clear:

- exactly one cluster disappears (and so N decreases by one).

Alternative Cluster Approaches

Consistent with the computational model presented previously, there are two basic approaches to implementing the cluster rule.

- Maintain a complete cluster partition and update the cluster partition *each time* any cell changes its state.
- Don't maintain a complete cluster partition. Instead, each time lightning strikes a forested cell construct (and then clear) only that cluster to which the cell belongs.

Research Issue 1 — Which approach is best, and what associated data structure and algorithm is required? For $X = 100$, $Y = 100$, $p = 0.1$, $q = 1/XY$ and 1000 simulated "years" of operation, how much more time efficient is one approach relate to the other?

Other Considerations

Research Issue 2 — Can the $O(XY)$ time complexity of the computational model be improved by partitioning the landscape into a *clear* list $\mathcal{C}$ and a *forested* list $\mathcal{F}$. Then, for each time step (year) the state changes can be generated as follows.

- The number of new forested cells is a $\mathit{Binomial}(|\mathcal{C}|, p)$ random variate.
- The number of lightning strikes is a $\mathit{Binomial}(|\mathcal{F}|, q)$ random variate.

Research Issue 3 — How does the frequency of *catastrophic* lightning strikes depend on p and q, and what is the steady-state value of

$$\lim_{t \to \infty} \left(\frac{\delta(1) + \delta(2) + \cdots + \delta(t)}{t} \right)$$

Research Issue 4 — How is $\delta(t)$ related to $N(t)/XY$ where $N(t)$ is the number of clusters at time t?