THE FUTURE...

• discrete univariate random variables

• multivariate distributions

• time series analysis

• stochastic processes
SOME APPLICATIONS

- Determine the exact distribution of $\bar{X}$ in the presence of skewed data or outliers to make statistical inferences about the population mean

- Generate *resources* such as tables, graphs, and charts which include means, critical points, variances, etc

- Modeling using the hazard function

- Identification of outliers in a dataset
EXAMPLES (cont’d)

\[X := \left[ [x \rightarrow x, x \rightarrow 2-x], [0, 1, 2],
[\text{"Continuous"}, \text{"PDF"}] \right];\]
EXAMPLES

\[ X := [[x \rightarrow 1], [0, 1], ["Continuous", "PDF"]]; \]
LIST OF SUBLISTS

• ordered functions that define the functional forms of distribution

• ordered list of real numbers defining endpoints of intervals in the first sublist

• indicates what distribution form the functions in the first sublist represent
THE COMMON DATA STRUCTURE

- ONE data structure that applies to all six functional forms

- referred to as the ”list of sublists”

- each random variable is presented in a list containing three sublists
MORE APPL ALGORITHMS

• generate random variates associated with a random variable

• plot the six functional forms of any fully-specified distribution

• provide basic statistical abilities

• determine common summary characteristics of random variates - mean, variance, etc
MORE APPL ALGORITHMS

• calculate the PDF of sums of independent random variables: $Z = X + Y$

• calculate the PDF of products of independent random variables: $Z = XY$

• calculate the PDF of the min and the max of independent random variables: $Z = \min\{X, Y\}$ and $Z = \max\{X, Y\}$

• calculate the PDF of the $r$th order statistic from a sample of $n$ iid random variables
APPL ALGORITHMS

• supply a common data structure for representing the distributions of univariate random variables

• convert amongst six functional forms (PDF, CDF, IDF, SF, HF, CHF)

• provide straightforward instantiation of well-known distributions (exponential, normal, uniform)

• determine the distribution of a simple transformation of a continuous random variable, \( Y = g(x) \)
APPL Overview

• MAPLE statistical procedures cannot be applied to PDFs - obtaining probability results impossible

• Karian and Tanis supplemented MAPLE with "about 130 procedures written specifically to promote explorations of probabalistic and statistical concepts"

• focus of APPL is constructing a probability package for random variables

• APPL is limited to univariate, continuous, independent random variables and the transformations and combinations that result
PROBABILITY TERMS

• *piecewise*: functions constructed by piecing together various functions

• *iid*: independent and identically distributed random variables

• specification:
  
  – *fully specified*: Uniform(0, 1)
  
  – *semi-specified*: Uniform(a, 1)
  
  – *unspecified*: Uniform(a, b)
MORE PROBABILITY FUNCTIONS

• Survivor Function (SF):
  
  \[- S_X(x) = 1 - F_X(x) \]

• Hazard Function (HF):
  
  \[- h_X(x) = \frac{f_X(x)}{S_X(x)} \]

• Cumulative Hazard Function (CHF):
  
  \[- H_X(x) = \int_{-\infty}^{x} h_X(s)ds \]
Inverse Distribution Function (IDF)

\[ F^{-1}(u) = \min_x \{ x : u < F(x) \} \]
Cumulative Distribution Function (CDF)

\[ F_X(x) = \int_{-\infty}^{x} f_X(s) \, ds \]
Probability Density Function (PDF)

\[ f(x) \]

\[ f_X(x) = \Pr(X = x) \]
APPL: BACKGROUND

• MAPLE based conceptual probability software package

• fills existing technological gap in probability theory

• successfully integrated into three classes at W&M
INTRODUCTION

• Axioms/Theorems:
  – name and describe frequent random variables
  – provide theoretical results of random variables
  – provide applied results for statistics applications

• Omission:
  – ability to automate naming, processing, and application of random variables (very tedious)
APPL:
A Probability Programming Language

Maj. Andrew G. Glen
Diane L. Evans
Lawrence M. Leemis