

$[Name, PhoneNumber]$

We first define two basic types—a *Name* type for names in the phone book, and a *PhoneNumber* type for phone numbers in the phone book.

$PhoneNumberType ::= Home \mid Work \mid Mobile$

Now we define a type for the type of a phone number, which can either be a *Home* number, a *Work* number, or a *Mobile* number.

$PhoneNumberEntry$ $number : PhoneNumber$ $type : PhoneNumberType$
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This schema defines the type *PhoneNumberEntry*, which represents the thing which one looks up in the phone book. An entry has two components: a number and a type.

$PhoneBook$ $namesToEntries : Name \mapsto \mathbb{P} PhoneNumberEntry$ $namesToPrimaryEntries : Name \mapsto PhoneNumberEntry$ $dom\ namesToEntries = dom\ namesToPrimaryEntries \tag{3.1}$ $\forall a_name : Name \mid a_name \in dom\ namesToPrimaryEntries \bullet$ $\quad namesToPrimaryEntries(a_name) \in namesToEntries(a_name) \tag{3.2}$ $\forall a_name : Name \mid a_name \in dom\ namesToEntries \bullet$ $\quad \forall name_entry_1, name_entry_2 : PhoneNumberEntry \mid$ $\quad\quad name_entry_1 \in namesToEntries(a_name) \wedge name_entry_2 \in namesToEntries(a_name) \bullet$ $\quad\quad name_entry_1.number = name_entry_2.number \Leftrightarrow name_entry_1 = name_entry_2$

This schema defines the type of a phone book. A phone book has two mappings—names to phone number entries and names to primary number entries. The *namesToEntries* mapping maps to a set of all the phone number entries associated with a name. *namesToPrimaryEntries* maps a name to one entry which is the primary.

The first constraint 3.1 states that the partial functions *namesToEntries* and *namesToPrimaryEntries* must have the same domains. That is, if a name has a set of phone numbers, then it must also have a primary number. Conversely, if a name has a primary number, then it must have a set of numbers.

The second constraint 3.2 states that for every name which has a primary number, it's associated primary number (found by applying the function) is in the set of numbers. In other words, you can't have a primary number which is not in the set of numbers.

The last constraint 3.3 ensures that you can't have two different phone number entries with different phone numbers. The specification says this in a funny way—it chooses two different entries from the set of entries, and then states that if their numbers match, then they must really be the same entry. So, first we bind *a_name* to a name in the phone book. Then we choose two name entries, and constrain them to be entries for the name we've bound. Then we state that the numbers are the same if and only if the entries are the same.