

Homework on Simple Analytic Model

1. The Sno-Cone Stand

Mr. K is the manager of a new Snowball Express sno-cone stand which will be opening for business. The stand is operated by a single attendant who is behind a window. People come up to the window, tell the attendant which flavor sno-cone they want, wait for the attendant to fill their order, then pay and leave. When there is more than one customer at the window, a line forms so that customers are served in the order that they arrived. People always buy one and only one sno-cone at \$3 each. The cost of materials to make sno-cones is negligible, so Mr. K considers all of the money taken in to be profit, except that the attendant has to be paid from the money received.

After interviewing several applicants for the attendant's position, Mr. K has narrowed his choice to two people: Fran and Bill. Fran can complete a transaction (take the order, make the sno-cone, take payment and make change) in 20 seconds on the average. Bill can perform the same job in 30 seconds, on the average. Fran is faster than Bill but demands \$12 per hour, whereas Bill will work for \$6 per hour.

Market research has revealed that, on the average, Mr. K can expect one customer per minute to come up to the window. The research also shows that if a customer comes to the window and there are already three people in line, then the customer will promptly turn about in a huff and storm off to Ben and Jerry's.

Studies of real sno-cone stands have further shown that, even though the average customer arrival rate is one per minute, usually the amount of time between customers is less than that. It's the occasional lull in business that makes the average time between customers longer than that which is usually observed. The same is true of the time it takes to make sno-cones. Thus, assume that the time between customer arrivals and the time it takes a person to make a sno-cone are exponentially distributed random values.

- Taking all of these facts into consideration, which person is the more cost-effective to hire: Fran or Bill?
 - Bill really wants this job, so he tells Mr. K that he will work for less than \$6/hour. What pay rate will he have to accept in order to be competitive with Fran?
 - Bill really wants this job but can't settle for less pay. If he sticks to the original pay rate of \$6/hour and goes to sno-cone school to improve his skills, how much will he have to improve (by how many seconds must he reduce his mean service time) in order to compete with Fran?
2. Consider a birth-death system with the following birth and death coefficients: $\lambda_k = (k + 2)\lambda$ for $k = 0, 1, 2, \dots$ and $\mu_k = k\mu$ for $k = 1, 2, 3, \dots$. All other coefficients are zero.
- Solve for P_k , the equilibrium probability of finding the system in state k . Be sure to express your answer explicitly in terms of λ , k , and μ only.
 - Find the average number of customers in the system.