Discrete-Event Simulation:

A First Course

Section 5.1: Next-Event Simulation

  0-13-142917-5

Section 5.1: Next-Event Simulation

- Making small modifications to our simple discrete-event simulations is non-trivial
  - Add feedback to `ssq2`
  - Add delivery lag to `sis2`
- Next-event simulation is a more general approach to discrete-event simulation
  - System state
  - Events
  - Simulation clocks
  - Event scheduling
  - Event list

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Definitions and Terminology – State

- The *state* of a system is a complete characterization of the system at an instance in time
  - Conceptual model - abstract collection of variables and how they evolve over time
  - Specification model - collection of mathematical variables together with logic and equations
  - Computational model - collection of program variables systematically updated
- **Example 5.1.1** State of ssq is number of jobs in the node
- **Example 5.1.2** State of sis is current inventory level

Definitions and Terminology - Events

- An *event* is an occurrence that may change the state of the system
- **Example 5.1.3** For ssq, events are arrivals or completion of a jobs
  - With feedback, the state *may* change
- **Example 5.1.4** For sis with delivery lag, events are demand instances, inventory reviews, and arrival of orders
- We can define artificial events
  - Statistically sample the state of the system
  - Schedule an event at a prescribed time

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Definitions and Terminology - Simulation Clock

- The *simulation clock* represents the current value of simulated time
- Discrete-event simulations lack definitive simulated time
  - As a result, it is difficult to generalize or embellish models
- **Example 5.1.5** It is hard to reason about `ssq2` because there are effectively two simulation clocks
  - Arrival times and completion times are not synchronized
- **Example 5.1.6** In `sis2`, the only event is inventory review
  - The simulation clock is integer-valued and we aggregate all demand

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Definitions and Terminology - Event Scheduling & Event List

- It is necessary to use a *time-advance mechanism* to guarantee that events occur in the correct order
- *Next-event* time advance is typically used in discrete-event simulation
- To build a *next-event* simulation:
  - construct a set of state variables
  - identify the event types
  - construct a set of algorithms that define state changes for each event type
- The *event list* is the data structure containing the time of next occurrence for each event type

Next-Event Simulation

Algorithm 5.1.1

1. **Initialize** - set simulation clock and first time of occurrence for each event type
2. **Process current event** - scan event list to determine most imminent event; advance simulation clock; update state
3. **Schedule new events** - new events (if any) are placed in the event list
4. **Terminate** - Continue advancing the clock and handling events until termination condition is satisfied

- The simulation clock runs asynchronously; inactive periods are ignored
- Clear computational advantage over *fixed-increment* time-advance mechanism

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Single-Server Service Node

- The state variable $l(t)$ provides a complete characterization of the state of a ssq

$$
\begin{aligned}
l(t) = 0 \quad &\iff \quad q(t) = 0 \quad \text{and} \quad x(t) = 0 \\
l(t) > 0 \quad &\iff \quad q(t) = l(t) - 1 \quad \text{and} \quad x(t) = 1
\end{aligned}
$$

- Two events cause this variable to change
  1. An arrival causes $l(t)$ to increase by 1
  2. A completion of service causes $l(t)$ to decrease by 1

Single-Server Service Node

- The initial state $l(0)$ can have any non-negative value, typically 0
- The terminal state can be any non-negative value
  - Assume at time τ arrival process stopped. Remaining jobs processed before termination
- Some mechanism must be used to denote an event impossible
  - Only store possible events in event list
  - Denote impossible events with event time of ∞

Single-Server Service Node

- The simulation clock (current time) is t
- The terminal ("close the door") time is τ
- The next scheduled arrival time is t_a
- The next scheduled service completion time is t_c
- The number in the node (state variable) is l

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Algorithm 5.1.2

Algorithm 5.1.2

```
l = 0;
t = 0.0;
t_a = GetArrival(); /* initialize the event list */
t_c = ∞;
while ((t_a < τ) or (l > 0)) {
    t = min(t_a, t_c); /* scan the event list */
    if (t == t_a) { /* process an arrival */
        l++;
        t_a = GetArrival();
        if (t_a > τ)
            t_a = ∞;
        if (l == 1)
            t_c = t + GetService();
    }
    else { /* process a completion */
        l−−;
        if (l > 0)
            t_c = t + GetService();
        else
            t_c = ∞;
    }
}
```

Program ssq3

- In `ssq3`, `number` represents $l(t)$ and structure `t` represents time
  - the event list `t.arrival` and `t.completion` (t_a and t_c from Algorithm 5.1.2);
  - the simulation clock `t.current` (t from Algorithm 5.1.2)
  - the next event time `t.next` ($\min(t_a, t_c)$ from Algorithm 5.1.2)
  - the last arrival time `t.last`
- Time-averaged statistics are gathered with the structure `area`
  - $\int_0^t l(s)\, ds$ evaluated as `area.node`
  - $\int_0^t q(s)\, ds$ evaluated as `area.queue`
  - $\int_0^t x(s)\, ds$ evaluated as `area.service`

World Views and Synchronization

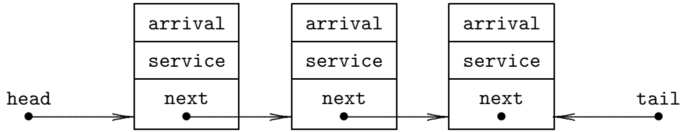
- Programs `ssq2` and `ssq3` simulate exactly the same system
- The two have different *world views*
  - `ssq2` naturally produces job-averaged statistics
  - `ssq3` naturally produces time-averaged statistics
- The programs should produce exactly the same statistics
  - To do so requires `rngs`

Model Extensions

Immediate Feedback

```
else { /* process a completion of service */
    if (GetFeedback() == 0) { /* this statement is new */
        index++;
        number--;
    }
```

- Alternate Queue Disciplines



Finite Service Node Capacity

Finite Service Node Capacity

```
if (t.current == t.arrival) {
    if (number < CAPACITY) {
        number++;
        if (number == 1)
            t.completion = t.current + GetService();
    }
    else
        reject++;
    t.arrival = GetArrival();
    if (t.arrival > STOP) {
        t.last = t.current;
        t.arrival = INFINITY;
    }
}
```

Random Sampling

- The structure of `ssq3` facilitates adding sampling
- Add a sampling event to the event list
  - Sample deterministically, every δ time units
  - Sample Randomly, every *Exponential*(δ) time units