Computer Science 616 - Stochastic Models in Computer Science
Fall 2007, Homework 3, due Sept 24 at noon

1. Chapter 2, Exercises 17, 18, 19, 20.

   17. Suppose that an experiment can result in one of r possible outcomes, the ith outcome having probability p_i, $i = 1, \ldots, r$ $\sum_{i=1}^{r} p_i = 1$. If n of these experiments are performed, and if the outcome of any of the n does not affect the outcome of the other $n-1$ experiments, then show that the probability that the first outcome appears x_1 times, the second x_2 times, and the rth x_r times is
   $$\frac{n!}{x_1!x_2!\ldots x_r!} p_1^{x_1} p_2^{x_2} \cdots p_r^{x_r}$$
   when $x_1 + x_2 + \ldots + x_r = n$. This is known as the multinomial distribution.

   18. Show that when $r = 2$ the multinomial reduces to the binomial.

   19. In Exercise 17, let X_i denote the number of times the ith outcome appears, $i = 1, \ldots, r$. What is the probability mass function of $X_1 + X_2 + \ldots + X_k$ (yes, the last index is $k \le r$)?

   20. A television store owner figures that 50 percent of the customers entering his store will purchase an ordinary tv set, 20 percent will purchase a HDMI tv set, and 30 percent will just be browsing. If five customers enter his store on a certain day, what is the probability that two customers purchase HDMI sets, one customers purchases an ordinary set, and two customers purchase nothing? 100

2. The density of X is
   $$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
   What is the value of c? What is the CDF of X? 20

3. The density of X is
   $$f(x) = \begin{cases} 10/x^2 & x > 10 \\ 0 & \text{otherwise} \end{cases}$$
   What is the CDF of X? What is the probability that X is greater than 20? 20

4. (Section 2, Exercise 40) Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability $1-p$. The winner of the series is the first to win four games. Find the expected number of games that are played, and evaluate this quantity when $p = 1/2$. 100

5. The definition of variance for a random variable X is $Var(X) = E[(X - E[X])^2]$. The expression inside the outermost expectation is the square of the distance of the value of X from its average. One reason for using the square is that it gives a "superlinear" importance to these distances, so that values of X far away from its average are given much weight. However, another reason is more mundane. Why doesn't it make any sense to define the variance as $E[X - E[X]]$? And how would you define a quantity analogous to the variance but in such a way that each distance of X from its average is given just a "linear" importance? 30