Finite Automata and Theory of Computation August 28, 2008

Handout 3: Homework 1

Professor: Moses Liskov

Due: September 4, 2008, in class

Problem 1

For this problem, all that is required is that you type up your problem set answers. You must, however, use appropriate mathematical typesetting to do so. A LaTeX template is available on the course web page to assist you. You must include the full text of all the problems in your homework (the purpose here is to make sure that you can typeset the appropriate symbols). *Note: this is only required for this problem set. In general, you are not requested to repdroduce the text of homework problems.*

Problem 2

The *power set* of a set S, denoted $\mathcal{P}(S)$, is defined as the set of all subsets of S. Give an example of two sets A and B such that $\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$; write out the members of $\mathcal{P}(A) \cup \mathcal{P}(B)$ and $\mathcal{P}(A \cup B)$.

Problem 3

Prove that for any sets A and B, $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

Problem 4

If A and B are sets, then the *set product* $A \times B$ is defined to be the set $\{(x, y) | x \in A \text{ and } y \in B\}$.

(a) For $A = \{0, 1\}$ and $B = \{1, 2\}$, determine $A \times B$.

(b) Does there exist a finite set S such that $S \times S = S$? if so, give S and $S \times S$. If not, prove it. *Hint: think about* $|S \times S|$.

Problem 5

Let $\Sigma = \{a, b\}$.

(a) Give a way to translate (positive, whole) numbers into strings over Σ in such a way that no two numbers translate to the same string. Are there strings your translation won't ever output?

(b) Using your translation from part (a), express the problem of determining whether or not a number is a prime number as a language over Σ.

Problem 6 (optional)

Let $\Sigma = \{a, b\}$. The "divisibility problem" is a mathematical problem; it is true of m, n if m divides into n evenly, with no remainder. Express the divisibility problem as a language over Σ.