

Name:

Section:

**Computer Science 243
Spring 2025
Homework 11**

Due: 11:59 p.m., Wednesday, 4/23/25

Points: 50

Answer the following questions and show your work. Your final submission must be completely your own work.

1. [6 points] List the ordered pairs in the relations below on $A = \{1, 2, 3, 4, 6\}$:
 - a. $R_1 = \{(a, b) \in R_1 \mid a \mid b\}$
 - b. $R_2 = \{(a, b) \in R_2 \mid a \cdot b \text{ is a power of } 2\}$
 - c. $R_3 = \{(a, b) \in R_3 \mid a \text{ and } b \text{ have the same number of letters}\}$
2. [4 points] List the relations in problem 1 that are
 - a. reflexive
 - b. symmetric
 - c. antisymmetric
 - d. transitive
3. [3 points] Find the following on the relations in problem 1:
 - a. $R_1 \cap R_2$
 - b. $R_1 - R_3$
 - c. $R_2 \circ R_3$
4. [5 points] For the relation $R = \{(0, 0), (0, 2), (1, 1), (1, 3), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$ on the set $\{0, 1, 2, 3\}$,
 - a. [1 point] write the matrix representing R
 - b. [1 point] draw the digraph of R
 - c. [3 points] list any needed closures (type of closure and closure itself) for R to make it an equivalence relation
5. [6 points] Which of the following are equivalence relations? For those that are not, list all missing properties along with an example that demonstrates each missing property. For parts (b) and (c), assume relations on the set of all people.
 - a. R_3 from problem 1(c)
 - b. $R_4 = \{(a, b) \in R_4 \mid a \text{ and } b \text{ share a common parent}\}$
 - c. $R_5 = \{(a, b) \in R_5 \mid a \text{ and } b \text{ live in different towns}\}$

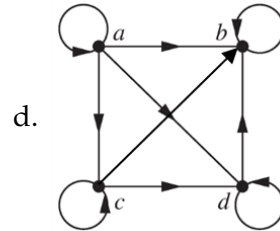
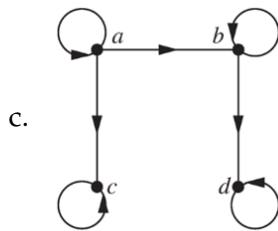
6. [4 points] Which of the following relations are posets? For those that are not, state any missing properties.

- [2 points] $\{(0,0), (1,1), (1,2), (1,3), (2,2), (3,1), (3,3)\}$ on the set $\{0,1,2,3\}$
- [1 point] $\{(0,0), (1,1), (2,2), (3,3)\}$ on the set $\{0,1,2,3\}$
- [1 point] $(a,b) \in R$ if a is taller than b on the set of all people

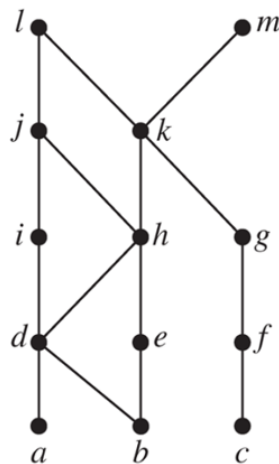
7. [8 points] Which of the following relations are posets on the set $\{a, b, c, d\}$? For those that are not, state any missing properties. For those that are, draw Hasse diagrams for each.

a.
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$



8. [14 points] Answer the following questions for the poset represented by the following Hasse diagram:



- Find the maximal elements. Find the minimal elements.
- What, if any, is the greatest element? The least element?
- Find all upper bounds of $\{a, b, c\}$.
- Find the least upper bound of $\{a, b, c\}$ if it exists.
- Find all lower bounds of $\{f, g, h\}$.
- Find the greatest lower bound of $\{f, g, h\}$, if it exists.
- Find the least upper bound of $\{c, e, j\}$ if it exists.