Name: **Section:**

Computer Science 243
Spring 2025
Homework 2

Due: 11:59 p.m., Monday, 2/10/25 **Points: 50**

Answer the following questions and show your work. Your final submission must be completely your own work.

1. [9 points each] Prove the following using logical equivalences (Chapter 1-I, Slides 40-42, or Tables 6 – 8 on pp. 27-28). Label each equivalence used.

   a. $\neg((p \rightarrow \neg q) \wedge q) \equiv p \vee \neg q$
   b. $(p \rightarrow (q \rightarrow p)) \wedge ((p \wedge \neg q) \rightarrow p) \equiv \mathbf{T}$

2. [3 points each] Consider the following, where the domain for x and y is $\{-1,1\}$:

   $p(x,y) : x = |y|$
   $q(x) : x < 0$

   Based on the above assignments, state the truth value for the each of following, along with your reasoning (i.e., list the values of x and/or y that make the statement true or false):

   a. $\forall x \exists y \; p(x,y)$
   b. $\exists y \forall x \; p(y,x)$
   c. $\forall x \forall y \; (p(x,y) \rightarrow q(x))$
   d. $\exists x \forall y \; (q(x) \rightarrow p(x,y))$

3. [3 points each] Given a domain of the integers for x and the negative integers for y, state the truth value of each of the following statements, along with your reasoning.

   a. $\forall x \exists y \; (x^2 = y)$
   b. $\forall x \exists y \; (x = y^2)$
   c. $\exists x \forall y \; (y^x = 1)$
   d. $\forall x \; (x \neq 0 \rightarrow \exists y \; (xy > 1))$

4. [2 points each] Given a domain of all people and the following predicates:
   $S(x)$: x is a student.
   $W(x)$: x lives in Williamsburg.
   $E(x,y)$: x emails y.

   translate the following English expressions into first-order logic statements:

   a. Some people living in Williamsburg are students.
   b. All students who live in Williamsburg email at least one other student.
   c. Nobody emails every student.
   d. No students who live in Williamsburg email themselves.