Name: **Section:**

Computer Science 243
Spring 2025
Homework 3

Due: 11:59 p.m., Friday, 2/14/25 **Points: 50**

Answer the following questions and show your work. Your final submission must be completely your own work.

1. [3 points each] Rewrite the following statements such that no negations appear outside a quantifier or expression involving a logical operators (show each step):

   a. $\neg \forall y \forall x \; P(x,y)$
   b. $\neg \exists y \forall x \; (\neg Q(y) \vee R(x,y))$
   c. $\neg \forall x \exists y \; (P(x,y) \rightarrow \neg Q(y))$

2. [8 points] Prove the following equivalence. Label each logical equivalence used in your proof.

$$\neg(\forall x \; (\neg p(x) \wedge (\forall y \; (q(y) \wedge \neg r(x,y))))) \equiv \exists x \; (\neg p(x) \rightarrow (\exists y \; (q(y) \rightarrow r(x,y))))$$

3. [10 points] Use rules of inference and/or logical equivalences to prove the following. Number your statements and label each rule of inference/logical equivalence used in your proof.

$$\begin{array}{l} (P \rightarrow Q) \wedge R \\ \neg(R \wedge Q) \\ S \rightarrow P \\ \hline \therefore \neg S \end{array}$$

4. [10 points] Use rules of inference and/or logical equivalences to prove the following. Number your statements and label each rule of inference/logical equivalence used in your proof.

$$\begin{array}{l} \exists x \; (q(x) \wedge r(x)) \\ \forall x \; (q(x) \rightarrow p(x)) \\ \hline \therefore \exists x \; (p(x) \wedge r(x)) \end{array}$$

5. [13 points] After converting the premises and conclusion below to predicate expressions, use the rules of inference and/or logical equivalences to prove the conclusion, where the domain of x is all people. Use $T(x)$, $D(x)$, and $C(x)$ as obvious predicates.

   Premises: Every teacher has a degree.
   Anyone who did not go to college is not a teacher.
   Someone is a teacher.

   Conclusion: Someone went to college and has a degree.

   Label each rule of inference used in your proof.