

Name:

Section:

**Computer Science 243  
Spring 2025  
Homework 5**

**Due: 11:59 p.m., Wednesday, 2/26/25**

**Points: 50**

Answer the following questions and show your work. Your final submission must be completely your own work.

1. [5 points] Prove or disprove the following, where the domain consists of all sets:  
 $\exists S (P(S) = \{S\})$   
that is, there exists a set,  $S$ , whose powerset is equal to  $\{S\}$ .
2. [12 points] State whether each function below is (i) one-to-one and (ii) onto. Provide reasoning for your answers. The domain for each function is the set of students in our CS 243 class.
  - a. a function that assigns a W&M ID number to a student (codomain: W&M ID numbers)
  - b. a function that matches students with their HW 1 papers (codomain: HW 1 papers in our CS 243 class with the assumption that every student submitted the assignment)
  - c. a function that assigns to a student a final letter grade, assuming an equal distribution of grades across 10-point intervals from 50 to 100 (codomain:  $\{A, B, C, D, F\}$ )
3. [6 points] Consider the functions below. Assume a domain and codomain of real numbers.
$$f(x) = -x + 5$$
$$g(x) = \pm\sqrt{|x|}$$
  - a. What is  $f^{-1}(x)$ ?
  - b. What is  $g \circ f$ ?
4. [8 points] Compute the value of each of the following:
  - a.  $\lfloor 1.4 \rfloor$
  - b.  $\lceil 1.4 \rceil$
  - c.  $\lfloor -2.5 \rfloor$
  - d.  $\lceil -2.5 \rceil$
  - e.  $\lfloor 4 \rfloor$
  - f.  $\lceil 4 \rceil$
  - g.  $\lfloor 3/2 + \lceil (1/2) \rceil \rfloor$
  - h.  $\lceil 3/2 + \lfloor (1/2) \rfloor \rceil$
5. [10 points] What are the first 6 terms ( $n = 0, 1, \dots, 5$ ) of the following sequences ( $n \geq 0$ )?
  - a. [6 points]  $a_n = (-2)^n$
  - b. [4 points]  $a_n = na_{n-1} + (a_{n-2})^2$ ,  $a_0 = 0$ ,  $a_1 = 1$

6. [9 points] Compute the following:

a.  $\sum_{i=1}^4 (3i - 1)$

b.  $\sum_{i=1}^3 \sum_{j=0}^2 (2i + 3)$

c.  $\sum_{i=2}^5 \sum_{j=1}^2 j^2$