Computer Science 243 Spring 2025 Homework 8

Due: 11:59 p.m., Wednesday, 4/2/25

Answer the following questions and show your work. Your final submission must be completely your own work.

1. [10 points] Prove the following by mathematical induction whenever *n* is a positive integer:

$$\sum_{i=1}^{n} (4i+2) = 2n(n+2)$$

- 2. [10 points] Prove by induction that $n^3 < n!$ for all integers $n \ge 6$. *Hint: Use Pascal's triangle where needed.*
- 3. [10 points] Prove the following by mathematical induction whenever *n* is a positive integer:

$$\sum_{i=1}^{n} (6i)^2 = 6n(n+1)(2n+1)$$

- 4. [6 points] Give a recursive definition of the following:
 - a. the set *S* of positive integers that are multiples of 4
 - b. the set *S* of positive integer powers of 5
 - c. the set *S* of positive integers congruent to 6 modulo 11
- 5. [14 points] For the sequence defined by $a_n = n(n-3)$ for $n \ge 3$:
 - a. [4 points] give a recursive definition of the sequence
 - b. [10 points] use an inductive proof to show that the recursive definition in (a) is equivalent to the original closed-form expression

Section:

Points: 50

Name: