Chapter 11 :: Functional Languages

Programming Language Pragmatics

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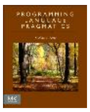

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Historical Origins

- The imperative and functional models grew out of work undertaken by Alan Turing, Alonzo Church, Stephen Kleene, Emil Post, etc. ~1930s
  - different formalizations of the notion of an algorithm, or *effective procedure*, based on automata, symbolic manipulation, recursive function definitions, and combinatorics
- These results led Church to conjecture that *any* intuitively appealing model of computing would be equally powerful as well
  - this conjecture is known as *Church's thesis*

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Historical Origins

- Turing's model of computing was the *Turing machine* a sort of pushdown automaton using an unbounded storage "tape"
  - the Turing machine computes in an imperative way, by changing the values in cells of its tape – like variables just as a high level imperative program computes by changing the values of variables

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Historical Origins

- Church's model of computing is called the *lambda calculus*
  - based on the notion of parameterized expressions (with each parameter introduced by an occurrence of the letter λ—hence the notation's name.
  - Lambda calculus was the inspiration for functional programming
  - one uses it to compute by substituting parameters into expressions, just as one computes in a high level functional program by passing arguments to functions

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Historical Origins

- Mathematicians established a distinction between
  - *constructive* proof (one that shows how to obtain a mathematical object with some desired property)
  - *nonconstructive* proof (one that merely shows that such an object must exist, e.g., by contradiction)
- Logic programming is tied to the notion of constructive proofs, but at a more abstract level
  - the logic programmer writes a set of *axioms* that allow the *computer* to discover a constructive proof for each particular set of inputs

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Functional Programming Concepts

- Functional languages such as Lisp, Scheme, FP, ML, Miranda, and Haskell are an attempt to realize Church's lambda calculus in practical form as a programming language
- The key idea: do everything by composing functions
  - no mutable state
  - no side effects

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Functional Programming Concepts

- Necessary features, many of which are missing in some imperative languages
  - 1st class and high-order functions
  - serious polymorphism
  - powerful list facilities
  - structured function returns
  - fully general aggregates
  - garbage collection

Functional Programming Concepts

- So how do you get anything done in a functional language?
  - Recursion (especially tail recursion) takes the place of iteration
  - In general, you can get the effect of a series of assignments

```
x := 0      ...
x := expr1  ...
x := expr2  ...
```

from f3(f2(f1(0))), where each f expects the value of x as an argument, f1 returns expr1, and f2 returns expr2

Functional Programming Concepts

- Recursion even does a nifty job of replacing looping

```
x := 0; i := 1; j := 100;
while i < j do
      x := x + i*j;
      i := i + 1;
      j := j - 1
end while
return x
```

becomes `f(0,1,100)`, where

```
f(x,i,j) == if i < j then
      f (x+i*j, i+1, j-1) else x
```

Functional Programming Concepts

- Thinking about recursion as a direct, mechanical replacement for iteration, however, is the wrong way to look at things
  - One has to get used to thinking in a recursive style
- Even more important than recursion is the notion of ***higher-order functions***
  - Take a function as argument, or return a function as a result
  - Great for building things

Functional Programming Concepts

- Lisp also has the following (which are not necessarily present in other functional languages)
  - homo-iconography
  - self-definition
  - read-evaluate-print
- Variants of LISP
  - Pure (original) Lisp
  - Interlisp, MacLisp, Emacs Lisp
  - Common Lisp
  - Scheme

Functional Programming Concepts

- Pure Lisp is purely functional; all other Lisps have imperative features
- All early Lisps dynamically scoped
  - Not clear whether this was deliberate or if it happened by accident
- Scheme and Common Lisp statically scoped
  - Common Lisp provides dynamic scope as an option for explicitly-declared *special* functions
  - Common Lisp now THE standard Lisp
    - Very big; complicated (The Ada of functional programming)

Functional Programming Concepts

- Scheme is a particularly elegant Lisp
- Other functional languages
  - ML
  - Miranda
  - Haskell
  - FP
- Haskell is the leading language for research in functional programming

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A Bit of Scheme

- As mentioned earlier, Scheme is a particularly elegant Lisp
  - Interpreter runs a read-eval-print loop
  - Things typed into the interpreter are evaluated (recursively) once
  - Anything in parentheses is a function call (unless quoted)
  - Parentheses are NOT just grouping, as they are in Algol-family languages
    - Adding a level of parentheses changes meaning

```
(+ 3 4) ⇒ 7
((+ 3 4))) ⇒ error
(the ' ⇒' arrow means 'evaluates to')
```

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A Bit of Scheme

- Scheme:
  - Boolean values `#t` and `#f`
  - Numbers
  - Lambda expressions
  - Quoting

```
(+ 3 4) ⇒ 7
(quote (+ 3 4)) ⇒ (+ 3 4)
'(+ 3 4) ⇒ (+ 3 4)
```

  - Mechanisms for creating new scopes

```
(let ((square (lambda (x) (* x x))) (plus +))
(sqrt (plus (square a) (square b))))
```

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A Bit of Scheme

- Scheme:
  - conditional expressions

```
(if (< x 0) (- 0 x))          ; if-then
(if  (< x y)  x  y)           ; if-then-else
(if (< 2 3) 4 5) ⇒ 4
(cond
 ((< 3 2) 1)
 ((< 4 3) 2)
 (else 3)) ⇒ 3
```

  - case selection

```
(case month
 ((sep apr jun nov)  30)
 (feb)  28)
 (else  31)
)
```

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A Bit of Scheme

- Scheme:
  - Imperative stuff
    - assignments
    - sequencing (begin)
    - iteration
    - I/O (read, display)

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A Bit of Scheme

- Scheme standard functions (this is not a complete list):
  - arithmetic
  - boolean operators
  - equivalence
  - list operators
  - symbol?
  - number?
  - complex?
  - real?
  - rational?
  - integer?

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A Bit of Scheme

- expressions
  - Cambridge prefix notation for all Scheme expressions:

```
(f x1 x2 … xn)

(+ 2 2)                ; evaluates to 4
(+ (* 5 4) (- 6 2))    ; means 5*4 + (6-2)
(define (Square x) (* x x)) ; defines a fn
(define f 120)         ; defines a global
```

  - Note: Scheme comments begin with ;

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- expression evaluation
  - three steps:
    1. Replace names of symbols by their current bindings.
    2. Evaluate lists as function calls in Cambridge prefix.
    3. Constants evaluate to themselves.

e.g.,

```
x                    ; evaluates to 5
(+ (* x 4) (- 6 2)); evaluates to 24
5                    ; evaluates to 5
'red                 ; evaluates to 'red
```

Source: Tucker & Noonan (2007)

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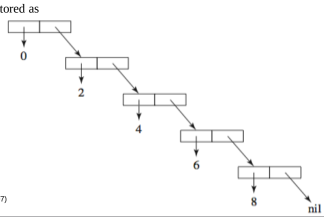
A Bit of Scheme

- lists
  - series of expressions enclosed in parentheses
  - represent both functions and data
  - empty list written as ()
  - e.g., (0 2 4 6 8) is a list of even numbers
    - stored as



Source: Tucker & Noonan (2007)

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A Bit of Scheme

- list transforming functions
  - using cons (construct):

```
(cons 8 ())                        ; gives (8)
(cons 6 (cons 8 ()))               ; gives (6 8)
(cons 4 (cons 6 (cons 8 ())))      ; gives (4 6 8)
(cons 4 (cons 6 (cons 8 9))) ; gives (4 6 8 . 9)
```

  - Note: the last element of a list should be a null list

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- list transforming functions
  - suppose we define the list evens to be (0 2 4 6 8), i.e., we write `(define evens '(0 2 4 6 8))`. Then,

```
(car evens)                 ; gives 0
(cdr evens)                 ; gives (2 4 6 8)
(cons 1 (cdr evens))        ; gives (1 2 4 6 8)
(null? '())                 ; gives #t, or true
(equal? 5 '(5))             ; gives #f, or false
(append '(1 3 5) evens) ; gives (1 3 5 0 2 4 6 8)
(list '(1 3 5) evens)   ; gives ((1 3 5) (0 2 4 6 8))
```

Note: the last two lists are different!

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- more on car/cdr

```
(car (cdr (evens))          ; gives 2
(cadr evens)                ; gives 2
(cdr (cdr (evens))          ; gives (4, 6, 8)
(cddr (evens)               ; gives (4, 6, 8)
(car '(6 8))                ; gives 6
(car (cons 6 8))            ; gives 6
(car '(8))                  ; gives 8
(cdr '(8))                  ; gives ()
```

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- defining functions

```
(define (name  arguments) function-body)

(define (min x y) (if (< x y) x y))
(define (abs x) (if (< x 0) (- 0 x) x))

define (factorial n)
  (if (< n 1) 1 (* n (factorial (- n 1)))
))
```

Note: be careful to match all parentheses

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- even simple tasks are accomplished recursively

```
((define (mystery1 alist)
 (if (null? alist) 0
        (+ (car alist) (mystery1 (cdr alist)))
))


(define (mystery2 alist)
 (if (null? alist) 0 (+ 1 (mystery2 (cdr
  alist)))
))
```

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- `subst` function

```
(define (subst y x alist)
 (if (null? alist) '())
    (if (equal? x (car alist))
        (cons y (subst y x (cdr alist)))
        (cons (car alist) (subst y x (cdr alist)))
)))
```

e.g., `(subst 'x 2 '(1 (2 3) 2))`
returns `(1 (2 3) x)`

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- `let` expressions allow simplification of function definitions by defining intermediate expressions

```
(define (subst y x alist)
   (if (null? alist) '()
  (let ((head (car alist)) (tail (cdr alist)))
     (if (equal? x head)
       (cons y (subst y x tail))
       (cons head (subst y x tail))
)))
```

Source: Tucker & Noonan (2007)

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A Bit of Scheme

- functions as arguments
  - **`mapcar`** applies the function to each member of a list

```
(define (mapcar fun alist)
    (if (null? alist) '()
        (cons (fun (car alist))
              (mapcar fun (cdr alist)))
))
```

e.g., if `(define (square x) (* x x))` then
`(mapcar square '(2 3 5 7 9))` returns
`(4 9 25 49 81)`

Source: Tucker & Noonan (2007)

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A Bit of Scheme
Example program - Symbolic Differentiation

- Symbolic Differentiation Rules

$$\frac{d}{dx}(c) = 0 \qquad c \text{ is a constant}$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx} \qquad u \text{ and } v \text{ are functions of } x$$

$$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(u/v) = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right)/v^2$$

Source: Tucker & Noonan (2007)

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A Bit of Scheme
Example program - Symbolic Differentiation

- Scheme encoding
  1. Uses Cambridge Prefix notation
     e.g., `2x + 1` is written as `(+ (* 2 x) 1)`
  2. Function `diff` incorporates these rules.
     e.g., `(diff 'x '(+ (* 2 x) 1))` should give an answer.
  3. However, no simplification is performed.
     e.g. the answer for (diff ‘x ‘(+ (* 2 x) 1)) is
     `(+ (+ (* 2 1) (* x 0)) 0)`
     which is equivalent to the simplified answer, 2

Source: Tucker & Noonan (2007)

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A Bit of Scheme
Example program - Symbolic Differentiation

- Scheme program

```
(define (diff x expr)
  (if (not (list? expr))
    (if (equal? x expr) 1 0)
     (let ((u (cadr expr)) (v (caddr expr)))
        (case (car expr)
          ((+) (list '+ (diff x u) (diff x v)))
          ((-) (list '- (diff x u) (diff x v)))
          ((*) (list '+ (list '* u (diff x v))
              (list '* v (diff x u))))
          ((/) (list 'div (list '- (list '* v (diff x u))
              (list '* u (diff x v)))
              (list '* u v)))
```

Source: Tucker & Noonan (2007)

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A Bit of Scheme
Example program - Symbolic Differentiation

- trace of the program

```
(diff 'x '(+ '(* 2 x) 1))
  = (list '+ (diff 'x '(*2 x)) (diff 'x 1))
  = (list '+ (list '+ (list '* 2 (diff 'x 'x))
              (list '* x (diff 'x 2)))
              (diff 'x 1))
   = (list '+ (list '+ (list '* 2 1) (list '* x (diff 'x 2)))
              (diff 'x 1))
   = (list '+ (list '+ '(* 2 1) (list '* x (diff 'x 2)))
              (diff 'x 1))
   = (list '+ (list '+ '(* 2 1) (list '* x 0))(diff 'x 1))
   = (list '+ (list '+ '(* 2 1) '(* x 0)(diff 'x 1))
   = (list '+ '('+ '(* 2 1) '(* x 0))(diff 'x 1))
   = (list '+ '('+ '(* 2 1) '(* x 0)) 0)
   = '(+ (+ (* 2 1) '(* x 0)) 0)
```

Source: Tucker & Noonan (2007)

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A Bit of Scheme
Example program - Simulation of DFA

- We'll invoke the program by calling a function called 'simulate', passing it a DFA description and an input string
  - The automaton description is a list of three items:
    - start state
    - the transition function
    - the set of final states
  - The transition function is a list of pairs
    - the first element of each pair is a pair, whose first element is a state and whose second element in an input symbol
    - if the current state and next input symbol match the first element of a pair, then the finite automaton enters the state given by the second element of the pair

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A Bit of Scheme
Example program - Simulation of DFA

```
(define simulate
  (lambda (dfa input)
    (letrec ((helper  ; note that helper is tail recursive,
                      ; but builds the list of moves in reverse order
                      (lambda (moves d2 i)
                        (let ((c (current-state d2)))
                          (if (null? i) (cons c moves)
                              (helper (cons c moves) (move d2 (car i)) (cdr i)))))))
      (let ((moves (helper '() dfa input)))
        (reverse (cons (if (is-final? (car moves) dfa)
                           'accept 'reject) moves))))))

;; access functions for machine description:
(define current-state car)
(define transition-function cadr)
(define final-states caddr)
(define is-final? (lambda (s dfa) (memq s (final-states dfa))))

(define move
  (lambda (dfa symbol)
    (let ((cs (current-state dfa)) (trans (transition-function dfa)))
      (list
       (if (eq? cs 'error)
           'error
           (let ((pair (assoc (list cs symbol) trans)))
             (if pair (cadr pair) 'error))) ; new start state
       trans                                ; same transition function
       (final-states dfa)))))               ; same final states
```

Figure 11.1 Scheme program to simulate the actions of a DFA. Given a machine description and an input symbol *i*, function `move` searches for a transition labeled *i* from the start state to some new state *s*. It then returns a new machine with the same transition function and final states, but with *s* as its "start" state. The main function, `simulate`, encapsulates a tail-recursive helper function that accumulates an inverted list of moves, returning when it has consumed all input symbols. The wrapper then checks to see if the helper ended in a final state; it returns the (properly ordered) series of moves, with `accept` or `reject` at the end. The functions `cadr` and `caddr` are defined as `(lambda (x) (car (cdr x)))` and `(lambda (x) (car (cdr (cdr x))))`, respectively. Scheme provides a large collection of such abbreviations.

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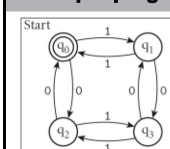
A Bit of Scheme
Example program - Simulation of DFA




```
(define zero-one-even-dfa
 '(q0                                                  ; start state
   (((q0 0) q2) ((q0 1) q1) ((q1 0) q3) ((q1 1) q0)   ; transition fn
    ((q2 0) q0) ((q2 1) q3) ((q3 0) q1) ((q3 1) q2))
   (q0)))                                              ; final states
```

Figure 11.2 DFA to accept all strings of zeros and ones containing an even number of each. At the bottom of the figure is a representation of the machine as a Scheme data structure, using the conventions of Figure 11.1.

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A Bit of OCaml

- OCaml is a descendent of ML, and cousin to Haskell, F#
  - "O" stands for objective, referencing the object orientation introduced in the 1990s
  - Interpreter runs a read-eval-print loop like in Scheme
  - Things typed into the interpreter are evaluated (recursively) once
  - Parentheses are NOT function calls, but indicate tuples

A Bit of OCaml

- Ocaml:
  - Boolean values
  - Numbers
  - Chars
  - Strings
  - More complex types created by lists, arrays, records, objects, etc.
  - (+ - * /) for ints, (+. -. *. /.) for floats
  - `let` keyword for creating new names

```
let average = fun x y -> (x +. y) /. 2.;;
```

A Bit of OCaml

- Ocaml:
  - Variant Types

```
type 'a tree = Empty | Node of 'a * 'a tree * 'a tree;;
```

  - Pattern matching

```
let atomic_number (s, n, w) = n;;
let mercury = ("Hg", 80, 200.592);;
atomic_number mercury;;  ⇒ 80
```

A Bit of OCaml

- OCaml:
  - Different assignments for references `':='` and array elements `'<-'`

```
let insertion_sort a =
        for i = 1 to Array.length a - 1 do
                let t = a.(i) in
                let j = ref i in
                while !j > 0 && t < a.(!j - 1) do
                        a.(!j) <- a.(!j - 1);
                        j := !j - 1
                done;
                a.(!j) <- t
        done;;
```

A Bit of OCaml
Example program - Simulation of DFA

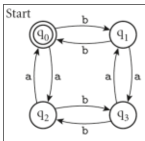
- We'll invoke the program by calling a function called 'simulate', passing it a DFA description and an input string
  - The automaton description is a record with three fields:
    - start state
    - the transition function
    - the list of final states
  - The transition function is a list of triples
    - the first two elements are a state and an input symbol
    - if these match the current state and next input, then the automaton enters a state given by the third element

A Bit of OCaml
Example program - Simulation of DFA

```
open List;;     (* includes rev, find, and mem functions *)

type state = int;;
type 'a dfa = {
  current_state : state;
  transition_function : (state * 'a * state) list;
  final_states : state list;
};;
type decision = Accept | Reject;;

let move (d:'a dfa) (x:'a) : 'a dfa =
  { current_state = (
      let (_, _, q) =
        find (fun (s, c, _) -> s = d.current_state && c = x)
             d.transition_function in
      q);
    transition_function = d.transition_function;
    final_states = d.final_states;
  };;

let simulate (d:'a dfa) (input:'a list) : (state list * decision) =
  let rec helper moves d2 remaining_input : (state option * state list) =
    match remaining_input with
    | [] -> (Some d2.current_state, moves)
    | hd :: tl ->
        let new_moves = d2.current_state :: moves in
        try helper new_moves (move d2 hd) tl
        with Not_found -> (None, new_moves) in
  match helper [] d input with
  | (None, moves) -> (rev moves, Reject)
  | (Some last_state, moves) ->
      ( rev (last_state :: moves),
        if mem last_state d.final_states then Accept else Reject);;
```

Figure 11.3 OCaml program to simulate the actions of a DFA. Given a machine description and an input symbol *i*, function `move` searches for a transition labeled *i* from the start state to some new state *s*. If the search fails, `find` raises exception `Not_found`, which propagates out of `move`; otherwise `move` returns a new machine with the same transition function and final states, but with *s* as its "start" state. Note that the code is polymorphic in the type of the input symbols. The main function, `simulate`, encapsulates a tail-recursive `helper` function that accumulates an inverted list of moves, returning when it has consumed all input symbols. The encapsulating function then checks to see if the helper ended in a final state; it returns the (properly ordered) series of moves, together with an `Accept` or `Reject` indication. The built-in `option` constructor (Example 7.6) is used to distinguish between a real state (`Some s`) and an error state (`None`).

A Bit of OCaml
Example program - Simulation of DFA




```
let a_b_even_dfa : char dfa =
  { current_state = 0;
    transition_function =
      [ (0, 'a', 2); (0, 'b', 1); (1, 'a', 3); (1, 'b', 0);
        (2, 'a', 0); (2, 'b', 3); (3, 'a', 1); (3, 'b', 2) ];
    final_states = [0];
  };;
```

Figure 11.4 DFA to accept all strings of as and bs containing an even number of each. At the bottom of the figure is a representation of the machine as an OCaml data structure, using the conventions of Figure 11.3.

Evaluation Order Revisited

- Applicative order
  - what you're used to in imperative languages
  - usually faster
- Normal order
  - like call-by-name: don't evaluate arg until you need it
  - sometimes faster
  - terminates if anything will (Church-Rosser theorem)

Evaluation Order Revisited

- In Scheme
  - functions use applicative order defined with lambda
  - special forms (aka macros) use normal order defined with syntax-rules
- A *strict* language requires all arguments to be well-defined, so applicative order can be used
- A *non-strict* language does not require all arguments to be well-defined; it requires normal-order evaluation

Evaluation Order Revisited

- Lazy evaluation gives the best of both worlds
- But not good in the presence of side effects.
  - delay and force in Scheme
  - delay creates a "promise"

High-Order Functions

- Higher-order functions
  - Take a function as argument, or return a function as a result
  - Great for building things
  - Currying (after Haskell Curry, the same guy Haskell is named after)
    - For details see Lambda calculus on CD
    - ML, Miranda, OCaml, and Haskell have especially nice syntax for curried functions

Functional Programming in Perspective

- Advantages of functional languages
  - lack of side effects makes programs easier to understand
  - lack of explicit evaluation order (in some languages) offers possibility of parallel evaluation (e.g. MultiLisp)
  - lack of side effects and explicit evaluation order simplifies some things for a compiler (provided you don't blow it in other ways)
  - programs are often surprisingly short
  - language can be extremely small and yet powerful

Functional Programming in Perspective

- Problems
  - –difficult (but not impossible!) to implement efficiently on von Neumann machines
    - lots of copying of data through parameters
    - (apparent) need to create a whole new array in order to change one element
    - heavy use of pointers (space/time and locality problem)
    - frequent procedure calls
    - heavy space use for recursion
    - requires garbage collection
    - requires a different mode of thinking by the programmer
    - difficult to integrate I/O into purely functional model

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