

Programming Languages

2nd edition
Tucker and Noonan

Chapter 15 Logic Programming

Q: How many legs does a dog have if you call its tail a leg?
A: Four. Calling a tail a leg doesn't make it one.
Abraham Lincoln

Contents

- 15.1 Logic and Horn Clauses
- 15.2 Logic Programming in Prolog
 - 15.2.1 Prolog Program Elements
 - 15.2.2 Practical Aspects of Prolog
- 15.3 Prolog Examples
 - 15.3.1 Symbolic Differentiation
 - 15.3.2 Solving Word Puzzles
 - 15.3.3 Natural Language Processing
 - 15.3.4 Semantics of Clite
 - 15.3.5 Eight Queens Problem

15.3.3 Natural Language Processing

- BNF can define natural language (e.g., English) syntax.
 - This was the original purpose of BNF when it was invented by Chomsky in 1957.
- A Prolog program can model a BNF grammar.
 - This was an original purpose of Prolog when it was designed in the 1970s.
- A Prolog list can model a sentence.
 - E.g., [the, giraffe, dreams]
- So running the program can parse a sentence.

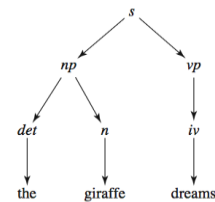
NLP Example

Consider the following BNF grammar and parse tree:

```

s → np vp
np → det n
   → iv
det → the
n → giraffe
   → apple
iv → dreams
tv → eats

```



Here, *s*, *np*, *vp*, *det*, *n*, *iv*, and *tv* denote "sentence," "noun phrase," "verb phrase," "determiner," "noun," "intransitive verb," and "transitive verb."

Prolog Encoding (naïve version)

```

s(X, Y) :- np(X, U), vp(U, Y).
np(X, Y) :- det(X, U), n(U, Y).
vp(X, Y) :- iv(X, Y).
vp(X, Y) :- tv(X, U), np(U, Y).
det([the | Y], Y).
n([giraffe | Y], Y).
n([apple | Y], Y).
iv([dreams | Y], Y).
tv([eats | Y], Y).

```

The first rule reads, "list X is a sentence leaving tail Y if X is a noun phrase leaving tail U and U is a verb phrase leaving tail Y."

Example Trace

```

?- s([the, giraffe, dreams], []).
Call: ( 7) s([the, giraffe dreams], []) ?
Call: ( 8) np([the, giraffe, dreams], _L131) ?
Call: ( 9) det([the, giraffe, dreams], _L143) ?
Exit: ( 9) det([the, giraffe, dreams], [giraffe, dreams]) ?
Call: ( 9) n([giraffe, dreams], _L131) ?
Exit: ( 9) n([giraffe, dreams], [dreams]) ?
Exit: ( 8) np([the, giraffe, dreams], [dreams]) ?
Call: ( 8) vp([dreams], []) ?
Call: ( 9) iv([dreams], []) ?
Exit: ( 9) iv([dreams], []) ?
Exit: ( 8) vp([dreams], []) ?
Exit: ( 7) s([the, giraffe, dreams], []) ?

```

Yes

The query asks, "can you resolve [the, giraffe, dreams] as an S, leaving tail []?"

The result is success.

7

Definite Clause Grammars (DCGs)

```
s --> np, vp.
np --> det, n.
vp --> iv.
vp --> tv, np.
det --> [the].
n --> [giraffe].
n --> [apple].
iv --> [dreams].
tv --> [eats].
```

This replaces
`s(X, Y) :- np(X, U), vp(U, V).`
 but it means the same thing.

Note: This form looks more like a series of BNF rules, and it's simple to write!

8

Generating Parse Trees

Terms and variables can be added to the DCG rules so that a parse tree can be generated from a query.

e.g., if we change

```
s --> np, vp.
```

to

```
s(s(NP, VP)) --> np(NP), vp(VP).
```

the variables NP and VP can capture intermediate subtrees.

When all rules are augmented in this way, the query

```
?- s(Tree, [the, giraffe, dreams], []).
```

delivers the parse tree as a parenthesized list:

```
Tree = s(np(det(the), n(giraffe)), vp(iv(dreams)))
```

9

15.3.4 Semantics of Clite

Program state can be modeled as a list of pairs. E.g.,

```
[[x, 1], [y, 5]]
```

Function to retrieve the value of a variable from the state:

```
get(Var, [[Var, Val] | _], Val).
get(Var, [_ | Rest], Val) :- get(Var, Rest, Val).
```

e.g.,

```
?- get(y, [[x, 5], [y, 3], [z, 1]], V).
V = 3
```

10

State transformation

Function to store a new value for a variable in the state:

```
onion(Var, Val, [[Var, _] | Rest], [[Var, Val] | Rest]).
onion(Var, Val, [Xvar | Rest], [Xvar | OState]) :-
onion(Var, Val, Rest, OState).
```

Note: second and third arguments denote the input and output states, resp.

E.g.,

```
?- onion(y, 4, [[x, 5], [y, 3], [z, 1]], S).
S = [[x, 5], [y, 4], [z, 1]]
```

11

Modeling Clite Abstract Syntax

| | |
|--------------------|-------------------------------------------|
| <i>Skip</i> | skip |
| <i>Assignment</i> | assignment(target, source) |
| <i>Block</i> | block([s1, ..., sn]) |
| <i>Loop</i> | loop(test, body) |
| <i>Conditional</i> | conditional(test, thenbranch, elsebranch) |
| <i>Expression</i> | |
| <i>Value</i> | value(val) |
| <i>Variable</i> | variable(id) |
| <i>Binary</i> | operator(term1, term2) |
| | where operator is one of plus, times, |
| | minus, div, lt, le, eq, ne, gt, ge |

12

Semantics of Statements

General form: minstruction(statement, inState, outState)

```
minstruction(skip, State, State).
minstruction(assignment(Var, Expr), InState, OutState) :-
mexpression(Expr, InState, Val),
onion(Var, Val, InState, OutState).
```

Notes: skip leaves the state unchanged.

For an **assignment**, the expression is first evaluated and the variable Val is instantiated.

Next, the target variable (Var) is assigned this value using the onion function.

Conditional, Loop, and Block

13

```
minstruction(conditional(Test, Thenbranch, Elsebranch), InState,
  OutState) :-
/* First, evaluate test. If it succeeds, Outstate is the meaning of
  Thenbranch in InState. Otherwise, it is the meaning of Elsebranch. */

minstruction(loop(Test, Body), InState, OutState) :-
/* First, evaluate test. If it succeeds, OutState is the meaning of loop with
  InState the meaning of Body in InState. Otherwise, OutState is the
  InState. */

minstruction(block([Head | Tail]), InState, OutState) :-
/* If Tail = [], then OutState is the meaning of Head. Otherwise, it is the
  meaning of block([Tail]) with InState the meaning of Head. */
```

Expression

14

```
mexpression(value(Val), _, Val).
mexpression(variable(Var), State, Val) :-
  get(Var, State, Val).
mexpression(plus(Expr1, Expr2), State, Val) :-
  mexpression(Expr1, State, Val1),
  mexpression(Expr2, State, Val2),
  Val is Val1 + Val2.
...
```

Note the creation of temporary variables Val1 and Val2, and the final assignment of their sum.

To Do:

15

1. Show that these definitions give 5 as the meaning of $y+2$ in the state $((x\ 5)\ (y\ 3)\ (z\ 1))$. I.e., show that

```
mexpression(plus(variable(y), value(2)), [[x,5], [y,3], [z,1]], Val)
```

```
...
```

```
Val = 5
```
2. Complete the definition of minstruction for conditionals, loops, and blocks.

15.3.5 Eight Queens Problem

16

A backtracking algorithm for which each trial move's:

1. Row must not be occupied,
2. Row and column's SW diagonal must not be occupied, and
3. Row and column's SE diagonal must not be occupied.

If a trial move fails any of these tests, the program backtracks and tries another. The process continues until each row has a queen (or until all moves have been tried).

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Q | | | | | | | |
| | Q | | | | | | |
| | | Q | | | | | |
| | | | Q | | | | |
| | | | | Q | | | |
| | | | | | Q | | |
| | | | | | | Q | |
| | | | | | | | Q |

Modeling the Solution

17

- Board is $N \times N$. Goal is to find all solutions.
- For some values of N (e.g., $N=2$) there are no solutions.
- Rows and columns use zero-based indexing.
- Positions of the queens in a list **Answer** whose i th entry gives the row position of the queen in column i , in reverse order.
 E.g., **Answer** = [4, 2, 0] represents queens in (row, column) positions (0,0), (2,1), and (4,2); see earlier slide.
- End of the program occurs when **Answer** has N entries or 0 entries (if there is no solution).
- Game played using the query:
 ?- queens(N , **Answer**).

Generating a Solution

18

```
solve(N, Col, RowList, _, _, RowList) :-
  Col >= N.
```

```
solve(N, Col, RowList, SwDiagList, SeDiagList, Answer) :-
  Col < N,
  place(N, 0, Col, RowList, SwDiagList, SeDiagList, Row),
  getDiag(Row, Col, SwDiag, SeDiag),
  NextCol is Col + 1,
  solve(N, NextCol, [Row | RowList], [SwDiag | SwDiagList],
    [SeDiag | SeDiagList], Answer).
```

Generating SW and SE Diagonals

19

```
getDiag(Row, Col, SwDiag, SeDiag) :-  
    SwDiag is Row + Col, SeDiag is Row - Col.
```

Generating a Safe Move

20

```
place(N, Row, Col, RowList, SwDiagList, SeDiagList, Row) :-  
    Row < N,  
    getDiag(Row, Col, SeDiag, SwDiag),  
    valid(Row, SeDiag, SwDiag, RowList, SwDiagList, SeDiagList).  
  
place(N, Row, Col, RowList, SwDiagList, SeDiagList, Answer) :-  
    NextRow is Row + 1,  
    NextRow < N,  
    place(N, NextRow, Col, RowList, SwDiagList, SeDiagList, Answer).
```

Checking for a Valid Move

21

```
valid(_, _, _, []).  
  
valid(TrialRow, TrialSwDiag, TrialSeDiag, RowList,  
    SwDiagList, SeDiagList) :-  
    not(member(TrialRow, RowList)),  
    not(member(TrialSwDiag, SwDiagList)),  
    not(member(TrialSeDiag, SeDiagList)).
```

Note: RowList is a list of rows already occupied.

Sample Runs

22

```
?- queens(1, R).  
R = [0].  
no  
?- queens(2, R).  
no  
?- queens(3, R).  
no  
?- queens(4, R).  
R = [2,0,3,1];  
R = [1,3,0,2];  
no
```

1x1 board has
one solution

2x2 and 3x3
boards have
no solutions

4x4 board has
two solutions