Computer Science 423
Fall 2024
Homework 3
My name

Due: beginning of class, Thursday, 9/26/2024

Answer the following questions and submit typeset solutions by the due date. As stated on the syllabus, any collaborators or outside sources must be listed under the corresponding problem. Further, your final submission must be completely your own work. Note: Draw state diagrams with no crossing arcs.

1. [5 points each] Let $\Sigma = \{a,b\}$. Draw state diagrams for NFAs that accept the following regular languages (given in the forms of regular expressions). Your NFAs should use as few states as possible. Note that "+" means one or more repetitions of a pattern.

   (a) $b(bab)^* \cup b$ (no more than 5 states)

   (b) $b^+ \cup (aa)^+$ (no more than 4 states)

   (c) $(a \cup b^+)a^+b^+$ (no more than 5 states)

   Collaborators:

2. [3 points each] Let R_1 and R_2 be any regular expressions. For each equivalence below, state whether it is True or False. No explanation necessary.

   (a) $(R_1 \cup R_2)^* = R_1^* \cup R_2^*$

   (b) $(R_1 R_2 \cup R_1)^* R_1 = R_1 (R_2 R_1 \cup R_1)^*$

   (c) $R_1^* (R_2 R_1^*)^* = (R_2^* R_1)^* R_2^*$

   (d) $(R_1 \cup R_2)^* R_2 = R_1^* R_2^+$

   (e) $R_2 (R_1 R_2 \cup R_2)^* R_1 = R_2 R_1 R_2^* (R_1 R_1^* R_2)^*$

   Collaborators:

3. [6 points each] For the languages given below, state whether each regular expression correctly defines the language. If the regular expression is incorrect, provide a counterexample of either a string that is generated by the regular expression but not in the language, or a string that is in the language but not generated by the regular expression.

   (a) Let $L_1 = \{w \in \{0,1\}^* | w$ has no substring of $00\}$.

      i. $(01^+)^* \cup (1^+0)^*$

      ii. $(1 \cup \varepsilon)(01^+)^*(0 \cup \varepsilon)$

      iii. $(1 \cup 01)^*(0 \cup \varepsilon)$

   (b) Let $L_2 = \{w \in \{0,1\}^* | w$ does not have 101 as a substring$\}$.

      i. $(\varepsilon \cup 01)(1 \cup 000^*)^*(\varepsilon \cup 10)$

      ii. $0^*(1^* \cup 1000^*1)^*0^*$

      iii. $(1 \cup 100^+1)^* \cup 0^*$

   Collaborators:

4. [4 points each] Let D be a language of strings that contains an even number of a's and an odd number of b's and does not contain the substring ab.

  (a) Give a simple regular expression for D.

  (b) Draw the state diagram for the DFA with no more than five states.

  Collaborators: