Computer Science 423
Fall 2024
Homework 8
My name and section

Due: beginning of class, Thursday, 11/7/2024 **Points: 50**

Answer the following questions and submit typeset solutions by the due date. As stated on the syllabus, any collaborators or outside sources must be listed under the corresponding problem. Further, your final submission must be completely your own work.

1. [2 points each] For each statement below, state whether it is True or False. No explanation necessary.

   (a) All Turing machines can be encoded into a unique binary string.

   (b) Every 2-tape nondeterministic Turing machine has an equivalent 1-tape deterministic Turing machine.

   (c) Every Turing-recognizable language is also Turing-decidable.

   (d) If A is Turing-recognizable and Turing machine M recognizes A, then there must exist a string $w \notin A$ for which M loops forever.

   (e) Hilbert's tenth problem concerning finding integral roots of polynomials is undecidable for all polynomials with degree $n \geq 0$.

   (f) An enumerator is a Turing machine with an extra output tape, on which any symbols, once written, can never be erased or modified.

   Collaborators:

2. [5 points each] Let $\Sigma = \{0,1\}$. For each of the following δ functions of Turing machines, draw the state diagram and describe the corresponding language $L(M)$. Note that the B stack symbol means blank.

   (a) $\delta(q_0,1) = (q_0,B,R)$, $\delta(q_0,0) = (q_1,B,R)$, $\delta(q_1,0) = (q_1,B,R)$, and $\delta(q_1,B) = (q_{accept},B,R)$.

   (b) $\delta(q_0,0) = (q_1,1,R)$, $\delta(q_1,1) = (q_2,0,L)$,, $\delta(q_2,1) = (q_0,1,R)$, and $\delta(q_1,B) = (q_{accept},B,R)$.

   Collaborators:

3. [6 points] Give the implementation-level description of a Turing machine that **decides** the following language

   $$L = \{w \in \{0,1\}^* \mid w \text{ contains more 1s than 0s}\}$$

   Collaborators:

4. [22 points] Consider the following δ functions of a Turing machine M (note that the B stack symbol means blank.):

$$\delta(q_0,0) = (q_1,B,R) \quad \delta(q_0,B) = (q_{accept},B,R)$$
$$\delta(q_1,0) = (q_1,0,R) \quad \delta(q_1,1) = (q_1,1,R) \quad \delta(q_1,B) = (q_2,B,L)$$
$$\delta(q_2,1) = (q_3,B,L)$$
$$\delta(q_3,0) = (q_3,0,L) \quad \delta(q_3,1) = (q_3,1,L) \quad \delta(q_3,B) = (q_0,B,R)$$

   (a) [6 points] Draw the state diagram for M.

   (b) [4 points] Infer the 7-tuple for M. You need not include the δ functions.

   (c) [4 points] Describe the language $L(M)$.

   (d) [4 points] Show a sample run on the single string of length 2 accepted by M. (Use notation similar to Sipster, p. 172.)

   (e) [4 points] Is $L(M)$ Turing-recognizable? Is $L(M)$ Turing-decidable?

   Collaborators: