Computer Science 423 Fall 2024 Homework 9 My name and section

Due: beginning of class, Thursday, 11/14/2024

Points: 50

Answer the following questions and submit typeset solutions by the due date. As stated on the syllabus, any collaborators or outside sources must be listed under the corresponding problem. Further, your final submission must be completely your own work.

- 1. [3 points each] For each language complement below, state whether it is (i) TD, (ii) TR but not TD, or (iii) non-TR. No explanation necessary.
 - (a) \overline{A}_{REX}
 - (b) \overline{A}_{CFG}
 - (c) \overline{HALT}_{TM}

Collaborators:

- 2. [2 points each] Let $A = \{0, 1, 2, ...\}$ and let $B = \{-1 \le x \le 1\}$, both with a universal domain of \mathbb{R} . State whether each of the following is Finite, Infinitely Countable, or Uncountable (select only one).
 - (a) *A*
 - (b) *B*
 - (c) A B
 - (d) B A
 - (e) $A \cap B$

Collaborators:

3. [12 points] List the properties (one-to-one, onto, correspondence) for (a) f(n) and (b) g(n). Assume a domain of $\{0, 1, 2, 3, 4\}$ and a codomain of $\{5, 6, 7, 8, 9\}$.

п	f(n)	n	g(n)
0	6	0	5
1	5	1	7
2	8	2	9
3	6	3	8
4	9	4	6

Collaborators:

4. [8 points] Let $L(M_1)$ and $L(M_2)$ be regular languages. Prove that the following is decidable:

 $U_{TM} = \{ < M_1, M_2, w > | w \text{ is accepted by either } M_1 \text{ or } M_2, \text{ but not both} \}$

(Hint: Use *A*_{DFA} as needed.)

Collaborators:

5. [11 points] Prove that the following is undecidable:

$$SUB_{TM} = \{ < M_1, M_2 > | L(M_1) \subseteq L(M_2) \}$$

(Hint: Reduce EQ_{TM} to two instances of SUB_{TM} .)

Collaborators: