CSCI 539 Algorithms

Homework 5

Due: November 20, 2001

1. In the closest-pair problem, let P be a point set with n points. Let X be the same point set sorted by x-coordinates. For points with the same x-coordinate, they are sorted by y-coordinates. Let Y be the same point set sorted by y-coordinates. For points with the same y-coordinate, they are sorted by x-coordinates. Recall that the closest-pair algorithm uses an imaginary vertical line to bisect X into X_L and X_R, which is easy to implement. As a result, Y is also partitioned into Y_L and Y_R, where Y_L and Y_R are the same point sets as X_L and X_R, respectively, sorted by y-coordinates. Describe in words a $O(n)$-time algorithm to create a partition of Y into Y_L and Y_R.

2. There are n points placed in a unit square. Show that the distance between the closest pair is $O(1/\sqrt{n})$. (Hint: Divide the unit square into $n-1$ smaller squares.)

3. Consider the algorithm $Select(L,k)$ that always chooses the first number in L as its pivot.

   (a) Describe an input of size n that makes the algorithm to achieve the $O(n^2)$ time bound.

   (b) Describe an input of size n that makes the algorithm to achieve the $O(n)$ time bound.

4. Consider the graph in Figure 9.82 on page 381 in MAW.

   (a) Find the minimum spanning tree for the graph using Prim's algorithm.

   (b) Find the minimum spanning tree for the graph using Kruskal's algorithm.