## CSCl312 Principles of Programming Languages

Semantics

Xu Liu

## Contents

7.1 Motivation
7.2 Expression Semantics
7.3 Program State
7.4 Assignment Semantics
7.5 Control Flow Semantics

### 7.1 Motivation

To provide an authoritative definition of the meaning of all language constructs for:

1. Programmers
2. Compiler writers
3. Standards developers

A programming language is complete only when its syntax, type system, and semantics are welldefined.

Semantics is a precise definition of the meaning of a syntactically and type-wise correct program.

Ideas of meaning:

- The meaning attached by compiling using compiler C and executing using machine M. Ex: Fortran on IBM 709.
- Axiomatize statements -- Chapter 12
- Statements as state transforming functions

This chapter uses an informal, operational model.

### 7.2 Expression Semantics



- $(a+b)-(c$ * $d)$
- Polish Prefix: - + a b * c d
- Polish Postfix: ab+cd*-
- Cambridge Polish: (- (+ a b) (* c d))

Infix uses associativity and precedence to disambiguate.

## Associativity of Operators

| Language | $+-* /$ | Unary $-* *$ | $==!=<\ldots$ |  |
| :--- | :--- | :--- | :--- | :--- |
| C-like | L | R |  | L |
| Ada | L | non | non | non |
| Fortran | L | R | R | L |

Meaning of: $\mathrm{a}<\mathrm{b}<\mathrm{c}$

## Precedence of Operators

| Operators | C-like | Ada | Fortran |
| :--- | :--- | :--- | :--- |
| Unary - | 7 | 3 | 3 |
| $* *$ |  | 5 | 5 |
| $* /$ | 6 | 4 | 4 |
| +- | 5 | 3 | 3 |
| $==!=$ | 4 | 2 | 2 |
| $\ll=\ldots$ | 3 | 2 | 2 |
| not | 7 | 2 | 2 |

## Short Circuit Evaluation

$a$ and $b$ evaluated as: if $a$ then $b$ else false
$a$ or $b$ evaluated as:
if $a$ then true else $b$

## Example

Node p = head;
while (p != null \& \& p.info != key)
$\mathrm{p}=\mathrm{p}$. next;
if ( $p==$ null $) / /$ not in list
else // found it

## Versus

boolean found $=$ false; while ( p != null \&\& ! found) \{ if ( p.info $==$ key)
found $=$ true; else

$$
\mathrm{p}=\mathrm{p} . \text { next }
$$

## Side Effect

A change to any non-local variable or I/O. What is the value of:

$$
\begin{aligned}
& i=2 ; b=2 ; c=5 ; \\
& a=b * i+++c^{*} i ;
\end{aligned}
$$

### 7.3 Program State

The state of a program is the collection of all active objects and their current values.

Maps:

1. The pairing of active objects with specific memory locations,
2. and the pairing of active memory locations with their current values.

The current statement (portion of an abstract syntax tree) to be executed in a program is interpreted relative to the current state.
The individual steps that occur during a program run can be viewed as a series of state transformations.

For the purposes of this chapter, use only a map from a variable to its value; like a debugger watch window, tied to a particular statement.
// compute the factorial of $n$
1 void main () \{
2 int n, i, f;
$3 \mathrm{n}=3$;
$4 \quad \mathrm{i}=1$;
$5 \mathrm{f}=1$;
6 while (i<n) \{
$7 \quad \mathrm{i}=\mathrm{i}+1$;
$8 \mathrm{f}=\mathrm{f} * \mathrm{i}$;
9 \}
10 \}


| // compute the factorial of n | n | i | f |
| :---: | :---: | :---: | :---: |
| 1 void main () \{ |  |  |  |
| 2 int $\mathrm{n}, \mathrm{i}, \mathrm{f}$; |  |  |  |
| $3 \mathrm{n}=3$; |  |  |  |
| $4 \mathrm{i}=1$; | 3 | undef | undef |
| $5 \mathrm{f}=1$; | 3 | 1 | undef |
| 6 while (i<n) \{ |  |  |  |
| $7 \quad \mathrm{i}=\mathrm{i}+1$; |  |  |  |
| $8 \mathrm{f}=\mathrm{f} * \mathrm{i}$; |  |  |  |
| 9 \} |  |  |  |
| 10 \} |  |  |  |


| // compute the factorial of n | n | i | f |
| :---: | :---: | :---: | :---: |
| 1 void main () \{ |  |  |  |
| 2 int $\mathrm{n}, \mathrm{i}, \mathrm{f}$; |  |  |  |
| $3 \mathrm{n}=3$; |  |  |  |
| $4 \mathrm{i}=1$; |  |  |  |
| $5 \mathrm{f}=1$; | 3 | 1 | undef |
| 6 while (i<n) \{ | 3 | 1 | 1 |
| $7 \quad \mathrm{i}=\mathrm{i}+1$; |  |  |  |
| $8 \mathrm{f}=\mathrm{f} * \mathrm{i}$; |  |  |  |
| 9 \} |  |  |  |
| 10 \} |  |  |  |


| // compute the factorial of n | n | i | f |
| :---: | :---: | :---: | :---: |
| 1 void main () \{ |  |  |  |
| 2 int n, i, f; |  |  |  |
| $3 \mathrm{n}=3$; |  |  |  |
| $4 \mathrm{i}=1$; |  |  |  |
| $5 \mathrm{f}=1$; |  |  |  |
| 6 while (i<n) \{ | 3 | 1 | 1 |
| $7 \quad \mathrm{i}=\mathrm{i}+1$; | 3 | 1 | 1 |
| $8 \mathrm{f}=\mathrm{f} * \mathrm{i}$; |  |  |  |
| 9 \} |  |  |  |
| 10 \} |  |  |  |


| // compute the factorial of n | n | i | f |
| :---: | :---: | :---: | :---: |
| 1 void main () \{ |  |  |  |
| 2 int $\mathrm{n}, \mathrm{i}, \mathrm{f}$; |  |  |  |
| $3 \mathrm{n}=3$; |  |  |  |
| $4 \mathrm{i}=1$; |  |  |  |
| $5 \mathrm{f}=1$; |  |  |  |
| 6 while (i<n) \{ |  |  |  |
| $7 \quad \mathrm{i}=\mathrm{i}+1$; | 3 | 1 | 1 |
| $8 \mathrm{f}=\mathrm{f} * \mathrm{i}$; | 3 | 2 | 1 |
| 9 \} |  |  |  |
| 10 \} |  |  |  |


| // compute the factorial of n | n | i | f |
| :---: | :---: | :---: | :---: |
| 1 void main () \{ |  |  |  |
| 2 int $\mathrm{n}, \mathrm{i}, \mathrm{f}$; |  |  |  |
| $3 \mathrm{n}=3$; |  |  |  |
| $4 \mathrm{i}=1$; |  |  |  |
| $5 \mathrm{f}=1$; |  |  |  |
| 6 while (i<n) \{ | 3 | 2 | 2 |
| $7 \quad \mathrm{i}=\mathrm{i}+1$; |  |  |  |
| $8 \mathrm{f}=\mathrm{f} * \mathrm{i}$; | 3 | 2 | 1 |
| 9 \} |  |  |  |
| 10 \} |  |  |  |

// compute the factorial of n
n
f
1 void main () \{
2 int n, i, f;
$3 \mathrm{n}=3$;
$4 \mathrm{i}=1$;
$5 \mathrm{f}=1$;

| 6 | while $(\mathrm{i}<\mathrm{n})\{$ | 3 | 2 | 2 |
| :--- | :---: | :--- | :--- | :--- |
| 7 | $\mathrm{i}=\mathrm{i}+1 ;$ | 3 | 2 | 2 |
| 8 |  | $\mathrm{f}=\mathrm{f} * \mathrm{i} ;$ | 3 | 3 |
| 9 | $\}$ |  | 3 | 3 |

10 \}

| // compute the factorial of n | n | i | f |
| :---: | :---: | :---: | :---: |
| 1 void main () \{ |  |  |  |
| 2 int $\mathrm{n}, \mathrm{i}, \mathrm{f}$; |  |  |  |
| $3 \mathrm{n}=3$; |  |  |  |
| $4 \mathrm{i}=1$; |  |  |  |
| $5 \mathrm{f}=1$; |  |  |  |
| 6 while (i<n) \{ | 3 | 3 | 6 |
| $7 \quad \mathrm{i}=\mathrm{i}+1$; |  |  |  |
| $8 \mathrm{f}=\mathrm{f} * \mathrm{i}$; |  |  |  |
| 9 \} |  |  |  |
| 10 \} | 3 | 3 | 6 |

### 7.4 Assignment Semantics

## Issues

- Multiple assignment
- Assignment statement vs. expression
- Copy vs. reference semantics


## Multiple Assignment

Example:

$$
\mathrm{a}=\mathrm{b}=\mathrm{c}=0 ;
$$

Sets all 3 variables to zero.
Problems???

## Assignment Statement vs. Expression

- In most languages, assignment is a statement; cannot appear in an expression.
- In C-like languages, assignment is an expression.
- Example: if $(\mathrm{a}=0) . . . / /$ an error
- while (*p++ = *q++) ; // strcpy
- while (ch = getc(fp)) ... // ???
- while ( $p=p->n e x t$ ) ... // ???


## Copy vs. Reference Semantics

- Copy: $\mathrm{a}=\mathrm{b}$;
- $\mathrm{a}, \mathrm{b}$ have same value.
- Changes to either have no effect on other.
- Used in imperative languages.
- Reference
- $\mathrm{a}, \mathrm{b}$ point to the same object.
- A change in object state affects both
- Used by many object-oriented languages.
public void add (Object word, Object number) \{
Vector set = (Vector) dict.get(word);
if (set $==$ null) \{ // not in Concordance
set $=$ new Vector( );
dict.put(word, set);
\}
if (allowDupl || !set.contains(number))
set.addElement(number);


### 7.5 Control Flow Semantics

To be complete, an imperative language needs:

- Statement sequencing
- Conditional statement
- Looping statement


## Sequence

s1 s2
Semantics: in the absence of a branch:

- First execute $\mathbf{s} 1$
- Then execute s2
- Output state of $s 1$ is the input state of $s 2$


## Conditional

IfStatement $\rightarrow$ if ( Expresion ) Statement
[ else Statement ]
Example:
if $(a>b)$
$z=a ;$
else

$$
z=b ;
$$

If the test expression is true, then the output state of the conditional is the output state of the then branch,
else the output state of the conditional is the output state of the else branch.

## Loops

WhileStatement $\rightarrow$ while (Expression) Statement
The expression is evaluated.
If it is true, first the statement is executed, and then the loop is executed again.

Otherwise the loop terminates.

## Semantic Interpretation

8.1 State Transformations and Partial Functions
8.2 Semantics of Clite
8.3 Semantics with Dynamic Typing
8.4 A Formal Treatment of Semantics

## Program State

The state of a program is the collection of all active objects and their current values.

Two maps in principle:

1. active objects $\longleftrightarrow$ memory locations,
2. active memory locations $\longleftrightarrow$ current values.

Only one map for our class:
variables $\longleftrightarrow$ current values

## Relations with program execution

The current statement (portion of an abstract syntax tree) to be executed in a program is interpreted relative to the current state.

The individual steps that occur during a program run can be viewed as a series of state transformations.

## Example (two maps):

Environment

- i,j at memory locations 154, 155

$$
\{<i, 154>,<j, 155>\}
$$

## Simplified to one

 map:$\{<\mathrm{i}, 13>,<\mathrm{j},-1>\}$

State

- i has value 13, $j$ has value -1

$$
\{\ldots,<154,13>,<155,-1>, \ldots\}
$$

### 8.1 State Transformations

Defn: The denotational semantics of a language defines the meanings of abstract language elements as a collection of state-transforming functions.

Defn: A semantic domain is a set of values whose properties and operations are independently wellunderstood and upon which the rules that define the semantics of a language can be based.

Example showing state transition
// compute the factorial of n
1 void main () \{
2 int n, i, f;
$3 \mathrm{n}=3$;
$4 \quad \mathrm{i}=1$;
$5 \mathrm{f}=1$;
6 while $(\mathrm{i}<\mathrm{n})$ \{
$7 \quad \mathrm{i}=\mathrm{i}+1$;
$8 \quad \mathrm{f}=\mathrm{f} * \mathrm{i}$;
9 \}
$10\}$

```
// compute the factorial of n
1 void main () {
2 int n, i, f;
3 n = 3;
{<n, undef> <i, undef> <f, undef>}
{<n,3> <i, undef> <f, undef>}
4 i = 1;
5 f=1;
w while (i < n) {
7 i= i + 1;
f f= f*i;
9 }
10 }
```

$$
\begin{aligned}
& \text { // compute the factorial of } n \\
& 1 \text { void main () \{ } \\
& 2 \text { int n, i, f; } \\
& 3 \mathrm{n}=3 \text {; } \\
& 4 \mathrm{i}=1 \text {; } \\
& 5 \mathrm{f}=1 \text {; } \\
& \{<\mathrm{n}, 3>\quad<\mathrm{i} \text {, undef }><\mathrm{f} \text {, undef }>\} \\
& \{<\mathrm{n}, 3>\quad<\mathrm{i}, 1><\mathrm{f}, \text { undef }>\} \\
& 6 \text { while }(\mathrm{i}<\mathrm{n}) \text { \{ } \\
& 7 \quad \mathrm{i}=\mathrm{i}+1 \text {; } \\
& 8 \quad \mathrm{f}=\mathrm{f} * \mathrm{i} \text {; } \\
& 9 \text { \} } \\
& 10 \text { \} }
\end{aligned}
$$

$$
\begin{aligned}
& \text { // compute the factorial of } \mathrm{n} \\
& \begin{array}{ll}
1 & \text { void main }()\{ \\
2 & \text { int } \mathrm{n}, \mathrm{i}, \mathrm{f} ; \\
3 & \mathrm{n}=3 ; \\
4 & \mathrm{i}=1 ; \\
5 & \mathrm{f}=1 ; \\
6 & \text { while }(\mathrm{i}<\mathrm{n})\{ \\
7 & \quad \mathrm{i}=\mathrm{i}+1 ; \\
8 & \quad \mathrm{f}=\mathrm{f} * \mathrm{i} ; \\
9 & \} \\
10 &
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \text { // compute the factorial of } \mathrm{n} \\
& \begin{array}{llll}
1 & \text { void main }()\{ \\
2 & \text { int } \mathrm{n}, \mathrm{i}, \mathrm{f} ; \\
3 & \mathrm{n}=3 ; \\
4 & \mathrm{i}=1 ; & \\
5 & \mathrm{f}=1 ; & & \\
6 & \text { while }(\mathrm{i}<\mathrm{n})\{ & \{<\mathrm{n}, 3> & <\mathrm{i}, 1><\mathrm{f}, 1>\} \\
7 & \mathrm{i}=\mathrm{i}+1 ; & \{<\mathrm{n}, 3> & <\mathrm{i}, 1><\mathrm{f}, 1>\} \\
8 & \mathrm{f}=\mathrm{f} * \mathrm{i} ; & & \\
\left.\begin{array}{l}
9
\end{array}\right\} & & \\
10 & \} & &
\end{array}
\end{aligned}
$$

$$
\left.\left.\begin{array}{l}
\text { // compute the factorial of } \mathrm{n} \\
\left.\begin{array}{lll}
1 & \text { void main }()\{ \\
2 & \text { int } \mathrm{n}, \mathrm{i}, \mathrm{f} ; \\
3 & \mathrm{n}=3 ; \\
4 & \mathrm{i}=1 ; \\
5 & \mathrm{f}=1 ; \\
6 & \text { while }(\mathrm{i}<\mathrm{n})\{ & \\
7 & \mathrm{i}=\mathrm{i}+1 ; & \{<\mathrm{n}, 3>
\end{array} \quad<\mathrm{i}, 1><\mathrm{f}, 1>\right\} \\
8
\end{array} \quad \mathrm{f}=\mathrm{f} * \mathrm{i} ; \quad \mathrm{n}, 3><\mathrm{i}, 2><\mathrm{f}, 1>\right\}\right\}
$$

$$
\begin{aligned}
& \text { // compute the factorial of } \mathrm{n} \\
& 1 \text { void main () \{ } \\
& 2 \text { int n, i, f; } \\
& 3 \mathrm{n}=3 \text {; } \\
& 4 \quad \mathrm{i}=1 \text {; } \\
& 5 \mathrm{f}=1 \text {; } \\
& 6 \text { while ( } \mathrm{i}<\mathrm{n} \text { ) \{ } \\
& 7 \quad \mathrm{i}=\mathrm{i}+1 \text {; } \\
& 8 \quad \mathrm{f}=\mathrm{f} * \mathrm{i} \text {; } \\
& 9 \text { \} } \\
& \begin{array}{lll}
\{<\mathrm{n}, 3> & <\mathrm{i}, 2> & <\mathrm{f}, 1>\} \\
\{<\mathrm{n}, 3> & <\mathrm{i}, 2> & <\mathrm{f}, 2>\}
\end{array} \\
& 10\}
\end{aligned}
$$

$$
\left.\begin{array}{l}
\text { // compute the factorial of } \mathrm{n} \\
\left.\begin{array}{llll}
1 & \text { void main }()\{ \\
2 & \text { int } \mathrm{n}, \mathrm{i}, \mathrm{f} ; \\
3 & \mathrm{n}=3 ; \\
4 & \mathrm{i}=1 ; \\
5 & \mathrm{f}=1 ; & & \\
6 & \text { while }(\mathrm{i}<\mathrm{n})\{ & \{<\mathrm{n}, 3> & <\mathrm{i}, 2>
\end{array}<\mathrm{f}, 2>\right\} \\
7
\end{array} \quad \mathrm{i}=\mathrm{i}+1 ; \quad\{<\mathrm{n}, 3><\mathrm{i}, 2><\mathrm{f}, 2>\}\right\}
$$

$$
\begin{aligned}
& \text { // compute the factorial of } \mathrm{n} \\
& 1 \text { void main () \{ } \\
& 2 \text { int n, i, f; } \\
& 3 \mathrm{n}=3 \text {; } \\
& 4 \mathrm{i}=1 \text {; } \\
& 5 \mathrm{f}=1 \text {; } \\
& 6 \text { while ( } \mathrm{i}<\mathrm{n} \text { ) \{ } \\
& 7 \quad \mathrm{i}=\mathrm{i}+1 \text {; } \\
& 8 \quad \mathrm{f}=\mathrm{f} \text { * } \mathrm{i} \text {; } \\
& 9 \text { \} } \\
& 10\} \\
& \{<\mathrm{n}, 3>\quad<\mathrm{i}, 3><\mathrm{f}, 6>\}
\end{aligned}
$$

### 8.2 C++Lite Semantics

State - represent the set of all program states
A meaning function M is a mapping:
M: Program $\rightarrow$ State
M: Statement x State $\rightarrow$ State
M: Expression $x$ State $\rightarrow$ Value

## Meaning Rule 8.1

The meaning of a Program is defined to be the meaning of the body when given an initial state consisting of the variables of the decpart initialized to the undef value corresponding to the variable's type.
decpart: the declaration part of the program.

## State M (Program p) \{

// Program = Declarations decpart; Statement body return $\mathrm{M}(\mathrm{p}$. body, initialState(p.decpart));
\}
public class State extends HashMap \{ ... \}

State initialState (Declarations d) \{
State state $=$ new State ( );
for (Declaration decl : d)
state.put(decl.v, Value.mkValue(decl.t));
\}
return state;

## Statements

M: Statement x State $\rightarrow$ State

Abstract Syntax

$$
\begin{aligned}
\text { Statement }= & \text { Skip } \mid \text { Block } \mid \text { Assignment } \mid \text { Loop } \mid \\
& \text { Conditional }
\end{aligned}
$$

State M(Statement s, State state) \{
if (s instanceof Skip) return $\mathrm{M}(($ Skip $) \mathrm{s}$, state $)$;
if (s instanceof Assignment) return $\mathrm{M}(($ Assignment $) \mathrm{s}$, state) ;
if (s instanceof Block) return $\mathrm{M}(($ Block $) \mathrm{s}$, state);
if (s instanceof Loop) return $\mathrm{M}(($ Loop $) \mathrm{s}$, state);
if (s instanceof Conditional) return $\mathrm{M}(($ Conditional $) \mathrm{s}$, state $)$;
throw new IllegalArgumentException( );

## Meaning Rule 8.2

The meaning of a Skip is an identity function on the state; that is, the state is unchanged.

State M(Skip s, State state) \{ return state;
\}
public class Skip extends Statement \{
public State meaning(State state) \{ return state;
$\}$

# public abstract class Statement \{ 

 public abstract State meaning(State state) ;\}

