Computer Science 243  
Summer 2017  
Homework 5

Due: Thursday, 7/27/2017

Answer the following questions and submit typeset solutions before the beginning of class on the due date. As stated in the Honor Code on the syllabus, any outside sources, including other students, must be listed under the corresponding problem. Further, your final submission must be completely your own work.

1. [10 points] Answer the following. Show all work to receive full credit.
   a. In a given state, license plates consist of either 4 upper-case letters followed by 2 digits [0-9] or 2 digits followed by 4 upper case letters. Furthermore, no license plate can start with a 0. How many possible license plates are possible?
   b. How many strings of five lowercase English letters can be created if (i) letters can be repeated, and (ii) if no letter can be repeated?

2. [10 points] How many bit strings of length 8
   a. end in 11?
   b. begin with 0 or end with 0?
   c. contain exactly 4 0’s?
   d. are palindromes?

3. [10 points] Suppose that a password for a computer system must have at least 6, but no more than 10, characters, where each character in the password is a lowercase English letter, and uppercase English letter, a digit, or one of six special characters *, @, #, $, %, or &.
   a. How many different passwords are possible for this system?
   b. How many different passwords are possible if the password is required to contain at least one special character?
   c. Using your answer in (a), how long would it take a hacker to try every possible password, assuming that it takes 1 ns to check a single password?

4. [10 points] Thirteen people on a softball team show up for a game.
   a. How many ways are there to choose 10 players to take the field?
   b. How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
   c. Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?
5. [10 points] Answer the following. Show all work to receive full credit.

a. At least how many students must be present in class to guarantee that there are at least 20 students of the same gender?

b. What is the minimum number of students, each of whom comes from one of the 50 U.S. states, who must be enrolled in a university to guarantee that there are at least 250 that come from the same state?

c. How many ordered pairs of integers \((a, b)\) are needed to guarantee that there are two ordered pairs \((a_1, b_1)\) and \((a_2, b_2)\) such that \(a_1 \pmod{5} = a_2 \pmod{5}\) and \(b_1 \pmod{5} = b_2 \pmod{5}\)?

6. [10 points] Answer the following. Show all work to receive full credit.

a. Find the expansion of \((x + y)^6\).

b. What is the coefficient of \(x^8y^9\) in the expansion of \((3x + 2y)^{17}\)?

7. [10 points] A bakery sells 6 different kinds of donuts: glazed, chocolate, plain, cinnamon, Bavarian custard, and blueberry. Assuming order of selection does not matter, how many ways are there to select

a. six donuts?

b. a dozen donuts with at least one of each kind?

c. two dozen donuts with no more than three chocolate?

8. [10 points] Answer the following.

a. How many different strings can be made from the word MISSISSIPPI, using all of the letters?

b. How many ways are there to distribute a deck of 52 cards to 4 players, with all players receiving the same number of cards?

9. [10 points] Use a proof by induction to show that the recurrence relation for the Tower of Hanoi, \(H_n = 2H_{n-1} + 1\) with \(H_1 = 1\), is equal to its closed-form solution, \(H_n = 2^n - 1\).

10. [10 points] Find a recurrence relation and initial conditions for the number of bit strings of length \(n\) that contain 3 consecutive 0’s. How many bit strings of length 8 contain 3 consecutive 0’s?