1. Describe an algorithm that uses a stack to determine whether a string is in the language $L$, where
   
   (a) $L = \{w \in \{A,B\}^* : w \text{ contains equal numbers of } A\text{'s and } B\text{'s}\}$
   
   (b) $L = \{w \in \{A,B\}^* : w \text{ is of form } A^nB^n \text{ for some } n \geq 0\}$

2. A deque is a data structure consisting of a list of items, on which the following operations are possible:
   
   - $\text{push}(x)$: Insert $x$ on the front end of the deque.
   - $\text{pop}()$: Remove the front item from the deque and return it.
   - $\text{inject}(x)$: Insert $x$ on the rear end of the deque.
   - $\text{eject}()$: Remove the rear item from the deque and return it.

   Describe routines to support the deque that take constant number of steps for each operation. You may use array-based or pointer-based implementation.

3. Solve the following recursive function by iterating, assuming $n = 4^k$ (i.e., $k = \log_4 n$) for some $k$.
   
   $$f(n) = \begin{cases} 
   1 & \text{if } n = 1 \\
   3f\left(\frac{n}{4}\right) + n & \text{if } n \geq 2 
   \end{cases}$$

4. Consider palindromes that consist only of lowercase letters from $\{a, b, \ldots, z\}$. Let $C(n)$ be the number of palindromes of length $n$. Write a recursive definition of $C(n)$. 