1. For each of the following program fragments, give an analysis of the running time. You may use summations to evaluate the running times of nested loops.

(a) \( \text{sum} = 0 \)
    \[
    \text{for } i = 1 \text{ to } n \\
    \quad \text{for } j = 1 \text{ to } i \cdot i \\
    \quad \quad \text{for } k = 1 \text{ to } j \\
    \quad \quad \quad \text{sum} ++
    \]

(b) \( \text{sum} = 0 \)
    \[
    \text{for } i = 1 \text{ to } n \\
    \quad \text{for } j = 1 \text{ to } i \cdot i \\
    \quad \quad \text{if } j \bmod i == 0 \\
    \quad \quad \quad \text{for } k = 1 \text{ to } j \\
    \quad \quad \quad \quad \text{sum} ++
    \]

2. For each pair of functions \((A, B)\) below, indicate whether \(A\) is \(O\), \(\Omega\), or \(\Theta\) of \(B\). Note that more than one of these relations may hold for a given pair; list all correct ones. No explanation is necessary.

(a) \((A, B) = ((\log n)^{10}, n^{0.01})\)
(b) \((A, B) = (\log(n!), \log(n^n))\)
(c) \((A, B) = (4^n, 2^n)\)
(d) \((A, B) = (n^{\log n}, 2^{\sqrt{\log n}})\)

3. Let \(A\) be an array of positive or negative integers of size \(n\), where \(A[1] < A[2] < \cdots < A[n]\). Design an \(O(\log n)\) algorithm to find an \(i\) such that \(A[i] = i\) provided such an \(i\) exists. Your algorithm should return 0 if such an \(i\) does not exist.

4. An array \(A[1 \ldots n]\) contains all the integers from 0 to \(n\) except one. It would be easy to determine the missing integer in \(O(n)\) time by using an auxiliary array \(B[0 \ldots n]\) to record which numbers appear in \(A\). In this problem, however, we cannot access an entire integer in \(A\) with a single operation. The elements of \(A\) are represented in binary, and the only operation we can use to access them is "fetch the \(j\)th bit of \(A[i]\)", which takes constant time. Show that if we use
only this operation, we can still determine the missing integer in $O(n)$ time. (Hint: Use divide-and-conquer. After one linear scan of the list, one can get a subproblem with size at most half of the original size.)