1. Determine the time complexity of Insertion Sort, Selection Sort, Bubble Sort, Heap Sort, Merge Sort, and Quick sort on sorted input, reversely sorted input, and input of identical elements. Assume that the input list has $n$ elements. Assume that in Quick Sort the pivot is the first element of the list to be sorted. You may provide the answers by filling out the following table.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Sorted</th>
<th>Reversely sorted</th>
<th>Identical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insertion Sort</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Selection Sort</td>
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<tr>
<td>Bubble Sort</td>
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<tr>
<td>Heap Sort</td>
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<tr>
<td>Merge Sort</td>
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<tr>
<td>Quick Sort</td>
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</tbody>
</table>

2. Suppose you are given a sorted list of $n-k$ elements followed by $k$ randomly order elements. How would you sort the entire list if

(a) $k = \Theta(1)$?
(b) $k = \Theta(\log n)$?
(c) $k = \Theta(\sqrt{n})$?

3. Professors Howard, Fine, and Howard have proposed the following “elegant” sorting algorithm:

```
Stooge-sort(A, i, j);
    if i+1 >= j then return;
    k = (j-i+1) div 3; //integer division
    Stooge-sort(A, i, j-k);
    Stooge-sort(A, i+k, j);
    Stooge-sort(A, i, j-k);
```

(a) Argue that $\text{Stooge-sort}(A, 1, n)$ correctly sorts $A[1...n]$ by using induction.
(b) Give a recurrence for the worst-case time of \texttt{Stooge-sort}(A, 1, n) and solve it in \(\Theta\).

(c) Is the algorithm any better than the existing \(\Theta(n \log n)\) sorting algorithms such as Heap Sort and Merge Sort? Do the professors deserve tenure because of this algorithm?

4. Let \(S_1, S_2, \ldots, S_k\) be sets of integers all in the range of 1 to \(n\). Assume the sum of the cardinalities of the \(S_i\)'s is \(n\). Describe an \(O(n)\) algorithm to sort all of the \(S_i\)'s separately. (Hint: You may want to use Radix Sort, a linear-time algorithm that sorts integers.)