Discrete-Event Simulation:

A First Course

Section 5.2: Next-Event Simulation Examples

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Outline

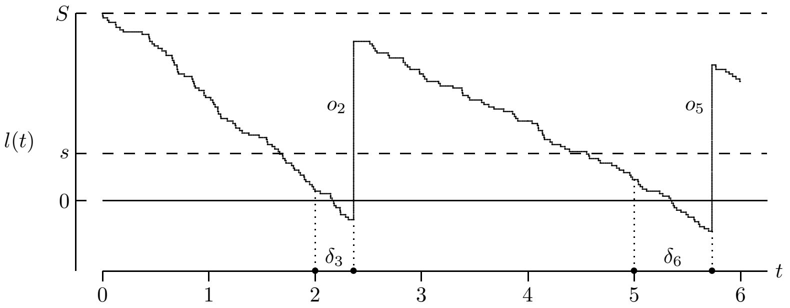
Two next-event simulation models will be developed

- Simple inventory system with delivery lag
- Multi-server service node

A Simple Inventory System with Delivery Lag

Two changes relative to `sis2`

- *Uniform*(0,1) lag between inventory review and order delivery
- More realistic demand model
  - Demand instances for a single item occur *at random*
  - Average rate is λ demand instances per time interval
  - Time between demand instances is *Exponential*$(1/\lambda)$



Comparison of Demand Models

- `sis2` used an *aggregate* demand for each time interval, generated as an *Equilikely*(10,50) random variate
  - Aggregate demand per time interval is random
  - Within an interval, time between demand instances is constant
  - Example: if aggregate demand is 25, inter-demand time is 0.04
- Now using *Exponential*$(1/\lambda)$ inter-demand times
  - Demand is modeled as an arrival process
  - Average demand per time interval is λ

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Specification Level: States and Notation

- The simulation clock is t (real-valued)
- The terminal time is τ (integer-valued)
- Current inventory level is $l(t)$ (integer-valued)
- Amount of inventory *on order*, if any, is $o(t)$ (integer-valued)
  - Necessary due to delivery lag
- $l(t)$ and $o(t)$ provide complete state description
- Initial state is assumed to be $l(0) = S$ and $o(0) = 0$
- Terminal state is assumed to be $l(\tau) = S$ and $o(\tau) = 0$
  - Cost to bring $l(t)$ to S at simulation end (with no lag) must be included in accumulated statistics

Specification Level: Events

Three types of events can change the system state

- A *demand* for an item at time t
  - $l(t)$ decreases by 1
- An inventory *review* at integer-valued time t
  - If $l(t) \geq s$, then $o(t)$ becomes 0
  - If $l(t) < s$, then $o(t)$ becomes $S - l(t)$
- An *arrival* of an inventory replenishment order at time t
  - $l(t)$ increases by $o(t)$
  - $o(t)$ becomes 0

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Algorithm 5.2.1, initialization

- Time variables used for event list:
  - t_d: next scheduled inventory *demand*
  - t_r: next scheduled inventory *review*
  - t_a: next scheduled inventory *arrival*
- ∞ denotes impossible events

Initialization Step of Algorithm 5.2.1

l = S;	/* initialize inventory level */
o = 0;	/* initialize amount on order */
t = 0.0;	/* initialize simulation clock */
t_d = GetDemand();	/* initialize event list */
t_r = t + 1.0;	/* initialize event list */
t_a = ∞;	/* initialize event list */

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Algorithm 5.2.1, main loop

Main Loop of Algorithm 5.2.1

```
while (t < τ) {
    t = min(t_d, t_r, t_a);  /* scan the event list */
    if (t == t_d) {          /* process an inventory demand */
        l--;
        t_d = GetDemand();
    }
    else if (t == t_r) {  /* process an inventory review */
        if (l < s) {
            o = S - l;
            δ = GetLag();
            t_a = t + δ;
        }
        t_r += 1.0;
    }
    else {                  /* process an inventory arrival */
        l += o;
        o = 0;
        t_a = ∞;
    }
}
```

Program sis3

- Implements Algorithm 5.2.1
- `t.demand`, `t.review` and `t.arrive` correspond to t_d, t_r, t_a
- State variables `inventory` and `order` correspond to $l(t)$ and $o(t)$
- `sum.hold` and `sum.short` accumulate the time-integrated holding and shortage integrals

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A Multi-Server Service Node

The single-server service node is extended to support multiple servers

- This is a natural generalization
- Multi-server service nodes have both practical and theoretical importance
- The event list size depends on the number of servers
  - For large numbers of servers, the event list data structure becomes important
- Extensions of the multi-server node (immediate feedback, finite capacity, non-FIFO) are left as exercises

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Conceptual Level

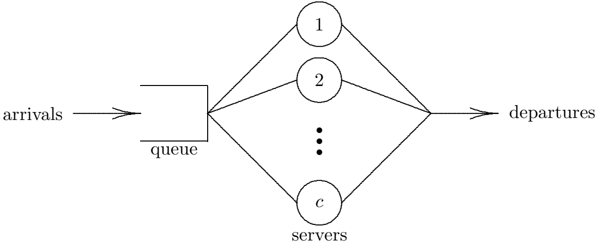
Definition 5.2.1: A *multi-server service node* consists of

- A single queue (if any)
- Two or more servers operating *in parallel*

At any instant in time,

- Each server is either *busy* or *idle*
- The queue is either *empty* or *not empty*
- If one or more servers is idle, the queue must be empty
- If the queue is not empty, all servers must be busy

Conceptual Level



- When a job arrives
  - If all servers are busy, the job enters the queue
  - Else an idle server is selected and the job enters service
- When a job departs a server
  - If the queue is empty, the server becomes idle
  - Else a job is removed from the queue, served by server
- Servers process jobs independently

Server Selection Rule

Definition 5.2.2: The algorithm used to select an idle server is called the *server selection rule*

- Common selection rules
  - Random selection: at random from the idle servers
  - Selection in order: lowest-numbered idle server
  - Cyclic selection: first available, starting after last selected (circular search may be required)
  - Equity selection: use longest-idle *or* lowest-utilized
  - Priority selection: choose the "best" idle server (modeler specifies how to dermine "best")
- Random, cyclic, equity: designed to achieve equal utilizations
- If servers are *statistically identical* and *independent*, the selection rule has *no effect* on average performance of the service node
- The *statistically identical* assumption is useful for mathematicians; unnecessary for discrete-event simulation

Specification Level: States and Notation

- Servers in a multi-server service node are called *service channels*
  - c is the number of servers (channels)
  - The *server index* is $s = 1, 2, \ldots, c$
- $l(t)$ denotes the number of jobs in the service node at time t
  - If $l(t) \geq c$, all servers are busy and $q(t) = l(t) - c$
  - If $l(t) < c$, some servers are idle
  - If servers are distinct, need to know which servers are idle
- For $s = 1, 2, \ldots, c$ define

  $x_s(t)$: the number of jobs in service (0 or 1) at server s at time t
- The complete state description is $l(t)$, $x_1(t)$, $x_2(t)$, $\ldots$, $x_c(t)$

$$q(t) = l(t) - \sum_{s=1}^{c} x_s(t)$$

Specification Level: Events

What types of events can change state variables $l(t), x_1(t), x_2(t), \ldots, x_c(t)$?

- An *arrival* at time t
  - $l(t)$ increases by 1
  - If $l(t) \leq c$, an idle server s is selected, and $x_s(t)$ becomes 1
  - Else all servers are busy
- A *completion of service* by server s at time t
  - $l(t)$ decreases by 1
  - If $l(t) \geq c$, a job is selected from the queue to enter service
  - Else $x_s(t)$ becomes 0

There are $c + 1$ event types

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Specification Level: Additional Assumptions

- The initial state is an empty node
  - $l(0) = 0$
  - $x_1(0) = x_2(0) = \cdots = x_c(0) = 0$
  - The first event must be an arrival
- The arrival process is turned off at time τ
  - The node continues operation after time τ until empty
  - The terminal state is an empty node
  - The last event is a completion of service
- For simplicity, all servers are independent and statistically identical
- Equity selection is the server selection rule

All of these assumptions can be relaxed

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Event List

0	t	x	arrival
1	t	x	completion of service by server 1
2	t	x	completion of service by server 2
3	t	x	completion of service by server 3
4	t	x	completion of service by server 4

- Can be organized as an array of $c + 1$ event types
- Field t: scheduled time of next occurrence for the event
- Field x: current *activity status* of the event
  - Superior alternative to using ∞ to denote impossible events
  - For 0^{th} event type, x denotes if arrival process is on or off
  - For other event types, x denotes if server is busy or idle
- For large c, consider alternate event-list structures (see section 5.3)

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Program `msq`

Implements this next-event multi-server service node simulation model

- State variable $l(t)$ is `number`
- State variables $x_1(t), x_2(t), \ldots, x_c(t)$ are part of the event list
- Time-integrated statistic $\int_0^t l(\theta)d\theta$ is `area`
- Array `sum` records for each server
  - the sum of service times
  - the number served
- Function `NextEvent` searches the event list to find the next event
- Function `FindOne` searches the event list to find the longest-idle server (because equity selection is used)

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