Household Electrical Load Scheduling Algorithms with Renewable Energy

Zhengrui Qin[1](✉) and Qun Li[2]

[1] School of Computer Science and Information Systems, Northwest Missouri State University, Maryville, MO 64468, USA
zqin@nwmissouri.edu

[2] Department of Computer Science, College of William and Mary, Williamsburg, VA 23185, USA
liqun@cs.wm.edu

Abstract. Efficient household electrical load scheduling benefits not only individual customers by reducing electricity cost but also the society by reducing the peak electricity demand and saving natural resources. In this paper, we aim to design efficient load scheduling algorithms for a household considering both real-time pricing policies and renewable energy sources. We prove that household load scheduling problem is NP-hard. To solve this problem, we propose several algorithms for different scenarios. The algorithms are lightweight and optimal or quasi-optimal, and they are evaluated through simulations.

1 Introduction

With the development of information and communication technologies, many smart meters have recently been deployed in the power grid, and more will be deployed in the near future. With these smart meters, time-varying pricing policy, which encourages customers to use power wisely, becomes practical. With time-varying pricing, consumers are motivated to shift their high-load appliances to off-peak periods, in which unit price is usually low. Furthermore, some houses may be equipped with a renewable energy system, such as a photovoltaic panel and/or a wind turbine, which also drives customers to seek smart load scheduling such that they can benefit more from renewable energy. In a word, efficient household electrical load scheduling benefits not only individual customers by reducing electricity cost but also society by reducing the peak electricity demand and saving natural resources.

In this paper, we formulate a general household load scheduling problem that considers both time-varying price and renewable energy sources. We show that finding an optimal solution is NP-hard. Then, we propose several load scheduling algorithms for different household appliances, with or without renewable energy. Different from previous work, our goal in this paper is to design generic lightweight and efficient algorithms such that our solution can handle all types of scheduling scenarios. We group household appliances into three categories:

S. Chellappan et al. (Eds.): WASA 2018, LNCS 10874, pp. 387–399, 2018.
https://doi.org/10.1007/978-3-319-94268-1_32

(1) schedulable and uninterruptable appliances; (2) schedulable and interruptable appliances; and (3) real-time appliances. For one appliance alone in each category, we propose optimal solutions. For multiple appliances of different categories, we propose efficient heuristic algorithms. Our main contributions in this paper are as follows:

- We formulate a generic household load scheduling problem which includes real-time pricing, with and without renewable energy.
- We design optimal algorithms to schedule any individual appliance. Especially, for electric water heater (EWH)-like appliances, we design an optimal algorithm using dynamic programming.
- We propose heuristic algorithms for multiple commonly used appliances.
- We evaluate our dynamic programming and heuristic algorithms via simulation studies. The results validate the efficiency and performance of our proposed algorithms.

2 Related Work

Time-varying price policies have been proposed since the last century. Examples are real-time pricing, time-of-use pricing, critical-peak pricing, and so on [1,2]. Given time-varying price, power cost saving via load scheduling has been extensively studied, such as [3–8]. In [5], the authors investigated residential load control in presence of a real-time pricing combined with inclining block rates. In [4], the authors discussed the load scheduling in households, buildings and warehouses, but they only considered uninterruptable appliances. In [3] and [7], the authors modeled the energy consumption scheduling with a carefully selected utility function, and they formulated to maximize the utility function minus electricity cost; however they consider a house as a whole and ignore the detailed appliance scheduling. In [6], Du and Lu have investigated the electrical load scheduling problem for a specific appliance, EWH. They solved the minimization problem with a multiloop heuristic algorithm, which, however, may not be optimal. In [8], Lu *et al.* have studied the scheduling for another specific appliance, HVAC (heating, ventilation and cooling).

Different from previous work, we tackle the household electrical load scheduling problem in a general sense by considering all types of appliances and renewable energy, and propose lightweight and efficient algorithms to solve it.

3 Problem Formulation

We group household appliances into three categories, based on their usage characteristics: (1) A_1, appliances that are schedulable but not interruptable, such as washer and dryer; (2) A_2, appliances that are both schedulable and interruptable, such as air conditioning unit (AC), and EWH; (3) A_3, real-time appliances, such as TV and microwave. Besides various appliances, a house may also have a renewable energy system, such as a photovoltaic system and/or a wind turbine system, which can store the renewable energy in a battery. We assume that the capacity of the battery that stores renewable energy is B kwh, and it can only be discharged up till a fraction c of its capacity, such as $c = 20\%$. We denote the renewable energy generating rate by G_t and the battery's discharge rate by F_t[1].

3.1 Constraints

For each appliance a, it usually has a preferred power range, denoted by $[r_a^{min}, r_a^{max}]$. For each schedulable appliances, there is a preferred time window, denoted by $[t_a^\alpha, t_a^\omega]$. Let $R_{a,t}$ denote the power of appliance a at time t, and let C_a denote the duration of a cycle for an appliance in A_1. Then general constraints for appliances in three categories are as follows:

$\forall a \in A_1$:

$$\begin{cases} r_a^{min} \le R_{a,t} \le r_a^{max}, & t \in [t_a, t_a + C_a] \subset [t_a^\alpha, t_a^\omega]; \\ R_{a,t} = 0, & \text{otherwise.} \end{cases} \tag{1}$$

$\forall a \in A_2$:

$$\begin{cases} r_a^{min} \le R_{a,t} \le r_a^{max} \text{ or } 0, & t \in [t_a^\alpha, t_a^\omega]; \\ R_{a,t} = 0, & t \notin [t_a^\alpha, t_a^\omega]. \end{cases} \tag{2}$$

$\forall a \in A_3$:

$$\begin{cases} r_a^{min} \le R_{a,t} \le r_a^{max}, & \text{if } a \text{ is on;} \\ R_{a,t} = 0, & \text{otherwise.} \end{cases} \tag{3}$$

Many appliances may have other specific constraints. For instance, the temperature of the water in an EWH cannot be too high or too low. We describe these specific constraints in a general form:

$$Specific\ Constraint(a), \forall a \in A. \tag{4}$$

Each house has a circuit breaker that automatically protects the household electricity system from overloading or short circuit. Thus, each house is constrained by the maximum power, denoted by R_{max}:

$$\sum_{a \in A} R_{a,t} - F_t \le R_{max} \tag{5}$$

Battery constraint:

$$cB \le B_0 + \sum_{t=0}^{i} (G_t - F_t)\delta t \le B \tag{6}$$

where B_0 is the energy already stored at the beginning of the first time slot.

[1] $B = G_t = F_t = 0$ if there is no renewable energy system.

3.2 Formulation

The ultimate goal for household load scheduling is to minimize the total electricity cost while fulfilling user's satisfaction. We assume the scheduling time domain is $[0, H]$, which is divided into m time slots with length of $\delta t = \frac{H}{m}$. The formulation is as follows.

$$Min : \sum_{t=1}^{m} (\sum_{a \in A} R_{a,t} - F_t) * \delta t * P_t + \sum_{t=0}^{H} \sum_{a \in A} D_{a,t} \tag{7}$$

Subject to: Constraints (1), (2), (3), (4), (5), and (6).

The first term in the objective function is the total electricity cost, and the second term is the penalty imposed by delayed services or unfulfilled services, where $D_{a,t}$ is the penalty of appliance a at time t.

4 Scheduling Algorithms

In this section, we will first show in a theorem that the problem formulated in Sect. 3 is NP-hard. Then we will design lightweight algorithms to solve the problem in different scenarios.

Theorem 1: The household load scheduling problem formulated in Sect. 3 is NP-hard.

Proof: We prove the theorem by the method of restriction, in which we show that a special case of the problem (less complicated than the original one) is NP-hard.

The special case is designed as follows. The renewable energy generating rate is G_0 all the time, and the discharge rate is F_0 all the time, where $F_0 \leq G_0$. Furthermore, there are only schedulable and uninterruptable appliances, which have the same usage duration, H/m, and the same scheduling window, $[0, H]$, where m is an integer. Finally, the power of each appliance is less than F_0. Since renewable energy is free, the cost minimization should first check whether renewable energy alone can satisfy the power demand. This is equivalent to packing all the jobs into m bins, with height of F_0 and width of H/m. Since all the usage durations are H/m, the solution with jobs scheduled across two bins is not better than the one without jobs across two bins. Therefore, it is equivalent to asking, whether we can pack all the appliances into less than or equal to m bins. This is obviously a bin packing problem, which is NP-hard. ■

Within a household, constraint Eq. (5) seldom takes effect; that is, the total consumption of a household usually does not surpass R_{max}. The circuit breaker seldom triggers unless there is a short circuit. Thus, we can schedule appliances individually in most cases, since they affect each other only when constraint Eq. (5) comes into effect. With this insight, we propose algorithms for individual appliances in each category, and ignore the second term in Eq. (7).

4.1 Algorithms for Different Scenarios

As the renewable energy is storable, the energy in the battery is non-descending if no renewable energy is used. We denote the amount of renewable energy available at time slot i, $i \in [1, m]$, by $RE_0(i)$, if no renewable energy is used.

Algorithm for One Appliance in A_1. The basic idea is to reserve as much renewable energy as possible at high-price periods for *each possible time window*. For each possible time window, we sort the price in descending order. For the highest-price time slot, we reserve as much renewable energy as needed and update renewable energy available for each time slot in this time window. Then we continue with the second-highest-price slot, and so on, until no more renewable energy can be scheduled.

Algorithm 1. Optimal scheduling for a washer alone.

Input: t_a^α, t_a^β, $C_a = i\delta t$.
Output: the start time t_0.

$t_0 = t_a^\alpha$;
$cost$ = a big number;
for $j \leftarrow t_a^\alpha$ **to** $t_a^\beta - i$ **do**
    $P_{temp,1:i} = sort(P_{j:j+i-1})$ // in descending order;
    $c = 0$;
    $RE = RE_0$;
    for $k \leftarrow 1$ **to** i **do**
        $t_* = \{t | t \in [j, j+i-1]\ \&\ P_t = P_{temp,k}\}$;
        $r = min\{RE(t_*), R_a * \delta t\}$;
        $RE = REupdate(t_*, RE, r)$;
        $c = c + (Ra * \delta t - r) * P_{t_*}$;
    if $c < cost$ **then**
        $cost = c$;
        $t_0 = j$;

The function $REupdate(t_*, RE, r)$ in Algorithm 1 is to update the available renewable energy at each time slot after r amount of renewable energy is reserved at time slot t_*. $REupdate(t_*, Re, r)$ returns an array with m elements. The function is as follows.

Algorithm for One Appliance in A_2. Here we take an EWH as a representative of schedulable and interruptable appliances. For an EWH alone, the formulation for time domain $[0, H]$ is as follows.

Min: $\sum_{t=0}^{H} R_{a,t} * \delta t * P_t$
Subject to:

$$\theta_t = f(R_{a,t}, \theta_{am}, \theta_{t-1}, \delta t) \tag{8}$$

Function REupdate(i,RE,r)

for $k \leftarrow i$ **to**,m **do**
  $RE(k) = RE(k) - r$;
for $k \leftarrow i-1$ **to** 1 **do**
  $RE(k) = min\{RE(k), RE(i)\}$;
return RE.

$$\theta_{low} \leq \theta_t \leq \theta_{high} \tag{9}$$

where f is the hot water thermal dynamics function, θ_{am} is the ambient temperature (the temperature of cold feed water), θ_t is the hot water temperature at time slot t, and θ_{low} and θ_{high} are the lower and upper limits of the comfort temperature band respectively. Here the penalty of dissatisfaction is not considered in the objective function, since the comfort temperature band, i.e., Eq. (9) is enforced.

EWH has been specifically studied by Du and Lu [6]. They have formulated the scheduling of EWH in a similar way, and solved it by a heuristic algorithm, which can achieve good performance but is not optimal. In the following, we aim to find the optimal solution through dynamic programming.

We equally divide the time domain $[t_0, H]$, such as a day, into m time slots, and divide the comfort temperature range $[\theta_{low}, \theta_{high}]$ into n domains. Then a grid mesh is formed in which x-axis is time t, y-axis is temperature θ, and each grid point (t_i, θ_j) means that hot water's temperature at time t_i is θ_j, $\forall 1 \leq i \leq m$ and $1 \leq j \leq n$, as shown in Fig. 1. The basic idea of our algorithm is to find the best scheduling for the transition from the starting point (t_0, θ_0) to another point (t_i, θ_j), where $t_0 < t_i$. The transition must go from the left side to the right side. For instance, Fig. 1 shows a transition from point P_1 to point P_2. We also need schedule the amount of renewable energy consumed during each transition. Similar to the temperature domain, we divide $RE_0(m)$ equally into l portions, and only an integer number of portions can be used during each time slot.

We use a function $Tr(t_0, \theta_0, t_i, \theta_j, r_i)$ to denote the cost of the optimal scheduling for the transition from the starting point (t_0, θ_0) to another point (t_i, θ_j), where r_i is the amount of renewable energy consumed so far at the point (t_i, θ_j). For the scheduling time domain $[t_0, H]$, the least cost is $Tr(t_0, \theta_0, H, \theta_H, r_m)$, where θ_H is the temperature of hot water at the end of the time domain. The dynamic programming method is shown in Algorithm 2.

Algorithm 2 outputs the least cost c_{min}. Once obtaining c_{min}, we can trace back to get the optimal path. Note that in the dynamic programming, the term $Tr(t_{i-1}, \theta_{j'}, t_i, \theta j, r)$ has to be calculated. The term represents the optimal cost from one point at time t_{i-1} (the beginning of time slot i) to another point at its next time t_i (the end of time slot i). We will calculate this term using an equivalent thermal parameter model [9,10]. According to this model, the temperature changing rate is non-linear with respect to time. The higher the initial temperature of EWH, the more difficult to increase the water temperature;

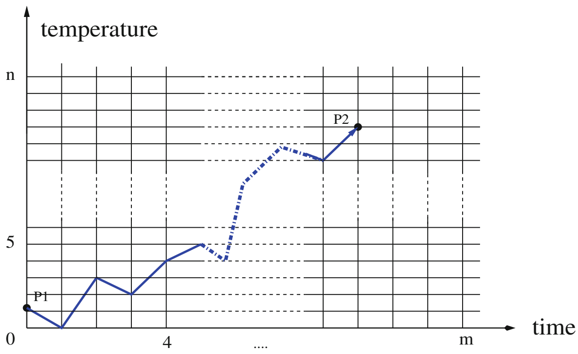

Fig. 1. A transition from point P_1 to point P_2. The time domain is divided into m slots and the temperature domain is divided into n slots. A valid transition should go from left to right.

Algorithm 2. Optimal heating scheduling for EWH for storable renewable energy.

Input: H, n, m, l, t_0, θ_0.
Output: The least cost, c_{min}.

$t_i = t_0 + i * H/m$, $\forall\ i \in [1, m]$;
$\theta_j = \theta_0 + j * (\theta_{high} - \theta_{low})/n$, $\forall\ j \in [0, n]$;
$\delta r = R(m)/l, \forall k \in [0, l]$;
for $i \leftarrow 2$ **to** m **do**
  for $j \leftarrow 0$ **to** n **do**
    for $k \leftarrow 1$ **to** $\lfloor R(i)/\delta r \rfloor$ **do**
      $r_i = \delta r * k$;
      $Tr(t_0, \theta_0, t_i, \theta_j, r_i) = min_{\forall j' \in [1,n], \forall r \in \{0,\ldots,r_i\}} \{Tr(t_0, \theta_0, t_{i-1}, \theta_{j'}, r_i - r) + Tr(t_{i-1}, \theta_{j'}, t_i, \theta_j, r)\}$;
$c_{min} = min_{\forall r \in [0,R]} \{Tr(t_0, \theta_0, t_m, \theta_0, r)\}$. // Find out the least cost;

---

the higher the initial temperature of EWH, the faster the water temperature decreases. We need to consider in total three types of transitions: (1) from (t_1, θ_1) to (t_2, θ_1), (2) from (t_1, θ_1) to (t_2, θ_2), where $\theta_1 < \theta_2$, and (3) from (t_1, θ_1) to (t_2, θ_2), where $\theta_1 > \theta_2$.

We assume that we have flat electricity prices within each time slot, which is usually the case in practice. For the first type of transition, the best scheduling should avoid high temperature during the transition, since high temperature is slow to reach and fast to vanish. Therefore, the best scheduling is switching on and off EWH alternatively, heating it for a moment and then letting it cool down to θ_1, and so on. For the second type of transition, the best scheduling should avoid temperature higher than θ_2. The last switch-on should directly increase the temperature to θ_2 without any cooling down. So the best scheduling has two sub-transitions: from (t_1, θ_1) to (t_x, θ_1) and from (t_x, θ_1) to (t_2, θ_2). The first sub-transition uses the same strategy as in the first type of transition, and the second sub-transition does not have any cooling down. Similar to the second type of transition, the third one is also split into two sub-transitions: from (t_1, θ_1) to (t_x, θ_2) and from (t_x, θ_2) to (t_2, θ_2). The first sub-transition does not have any heating, and the second sub-transition uses the same strategy as in the first type of transition.

Algorithm for One Appliance in A_3. In this case, the algorithm is very simple. The optimal solution is to assign as much renewable energy as needed to the time slots in price descending order.

Algorithm for All Appliances. As proved in the beginning of this section, this problem is NP-hard. In previous subsections, we propose algorithms for different types of appliances individually. Here we consider all types of appliances together. For a household, constraint (5) usually does not take effect; that is, the consumption of a household does not surpass R_{max}. We propose a heuristic algorithm, which schedules appliances according to their priorities. We assume the real-time appliances have the highest priority, followed by schedulable but uninterruptable appliances, then by the schedulable and interruptable appliances. We refer to this algorithm still as Algorithm MA.

4.2 Complexity and Performance

Our goal in this paper is to provide lightweight algorithms for an NP-hard problem—the household load scheduling problem. All algorithms proposed in this section are indeed lightweight. We list the complexity of all algorithms in Table 1, which are all polynomial.

Table 1. The complexity of proposed algorithms.

Scenarios	One in A_1	One in A_2	One in A_3	Multiple appliances
Complexity	$O(m)$	$O(mn^2l^2)$	$O(1)$	$O(mn^2l^2)$

As to performance, algorithms for each individual appliance are optimal or quasi-optimal (in the dynamic program, we use discretized temperature and renewable energy values), no matter whether there is renewable energy. As to the heuristic algorithms for multiple appliances of different types with renewable energy, they are not optimal. We cannot even derive a theoretical approximation bound for them, since no such bound exists as shown in the next subsection.

4.3 Discussion

Here we show that with renewable energy, no α-approximation algorithm exists.

Suppose we are given n appliances, each with a duration of C_j, during which it has a power of R_j. To simplify things, we assume all appliances are uninterruptable, and that the appliances can be scheduled at any time from 0 to T. The cost of one unit of energy at time t is P_t. The goal is to assign each appliance a time interval of length C_j such that the total cost of the energy consumed is minimized.

Claim 1: There exists no α-approximation algorithm for the household scheduling problem for any α.

Proof: We prove it by reduction. The reduction is from 2-partition. Given n items with sizes $a_1, \ldots, a_n$, can they be partitioned into two subsets of equal size? We set $RE_i = \frac{1}{2}\sum_{j=1}^{n} a_j$ for $i = 1, 2$, and $RE_i = 0$ otherwise, and create n jobs with $R_j = a_j$ and $C_j = 1$. Then a schedule of 0 cost corresponds to a partition, and any α-approximation must also be a schedule of 0 cost (to get any approximation guarantees, one has to exclude the possibility of a 0 cost schedule). ■

5 Evaluation

In this section, we will evaluate our algorithms for different appliances under different scenarios via simulations. We mainly evaluate our dynamic programming based algorithm (referred to as *Price-Driven Dynamic Programming algorithm* (PDDP)) for schedulable and interruptable appliances, and compare the cost with those obtained from existing schemes. We also evaluate the approximation ratio for the proposed heuristic algorithms under different scenarios.

5.1 Performance of PDDP Algorithm for an EWH

Here we will evaluate our algorithm, PDDP, for EWH with simulation. In [6], the authors compared their algorithm (we call it *Du & Lu's algorithm*) with the non-price-sensitive algorithm and the transactive control algorithm [11], and concluded that their algorithm outperforms the other two. Hereby, we will compare PDDP algorithm with Du & Lu's algorithm, the best of the three algorithms evaluated in [6].

We evaluate our PDDP algorithm with Ameren's residential real time pricing program [12]. Ameren's real time pricing program is an electric supply rate option in which customers pay electricity supply rates that vary by the hour. With this program, hourly prices for the next day are set the night before and can be communicated to customers so that they can determine the best time of day to use major appliances. Figure 2 shows the real-time price for Nov 3 (Sunday), Nov 4 (Monday), and Nov 7 (Thursday), 2013.

Furthermore, according to the past usage profile [13], we can forecast the daily hot water usage for a household. Based on [13], we can obtain hourly fraction of daily hot water draw. Suppose a typical household has 5 persons who consume about 180 gallons hot water daily. Thus, we obtain the hot water daily usage rate for a typical house as shown in Fig. 3.

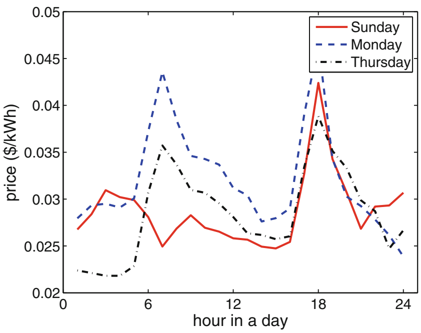

Fig. 2. Daily price.

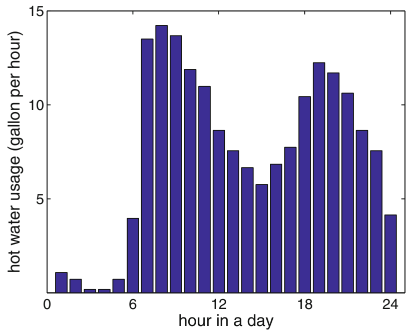

Fig. 3. Daily water usage forecast.

We simulate PDDP algorithm and Du & Lu's algorithm for a whole day. At the beginning, the temperature in the water tank is at the lower limit of the temperature band; at the end, the temperature goes back its original value. Figure 4 shows the EWH temperature profile using water usage in Fig. 3 and real price on Monday in Fig. 2. As we can see, Du & Lu's algorithm has high-temperature intervals, while PDDP only has medium-temperature intervals. Furthermore, the locations of intervals above the lower temperature bound are different. The high-temperature intervals of Du & Lu's algorithm reside in low-price periods, while those of PDDP are not necessarily in low-price periods. These differences contribute to 4 cent saving of electricity. Of course, the savings are different for different days due to different daily real time price. We conduct the simulation for the whole November, 2013, by using the water usage in Fig. 3 and Ameren's real time price. Figure 5 shows the daily cost for both algorithms (upper panel) and the daily saving (bottom panel). The daily saving can be as large as 24 cents and as small as 0.5 cents as well, but never be negative. Therefore, PDDP always outperforms Du & Lu's algorithm. On average, the daily cost for EWH is $\$0.913 \pm 0.085$, and that for Lu and Du's algorithm is $\$0.968 \pm 0.084$. The daily saving is \$0.055, accounting for about 6% saving; and the monthly saving is about \$1.65.

5.2 Approximation Ratios of Algorithm for Multiple Appliances

Since Algorithm MA, algorithm for multiple appliances, is heuristic, we evaluate its performance using the approximation ratio, denoted by ar, which is defined by the following equation:

$$ar = \frac{\text{the cost from } Alg.MA}{\text{the optimal cost}} \tag{10}$$

We set up two simulations. The first one has three appliances, one of each type; namely a stove, a washer, and an EWH. Note that Algorithm MA schedules appliances one by one based on their priorities. By running Algorithm MA, we can get the approximate cost. We also get the optimal solution by a variation

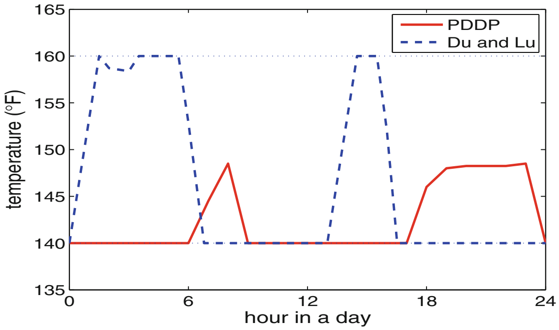

Fig. 4. Hot water temperature (°F) profiles for *PDDP algorithm* and *Du & Lu's algorithm.*

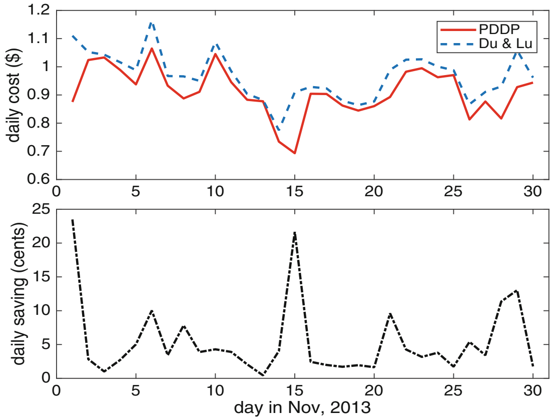

Fig. 5. Daily hot water cost comparison in Nov, 2013, for a typical household.

of our dynamic programing. Then we can calculate *ar* according to Eq. (10). The second simulation has an EWH, an AC, a washer and some real-time appliances, by considering that a household usually has two main schedulable and interruptable appliances, i.e., AC and EWH. In this simulation, we obtain the optimal cost through a brute-force search with dynamic programming. In both simulations, for a washer, we randomly pick a scheduling time window $[t_a^\alpha, t_a^\beta]$; for an EWH, we still use daily hot water profile in [13], with some random noise; for an AC, we set temperature range as [72,75]° F.

For the renewable energy, we consider the solar power generated by a photovoltaics (PV) array. We utilize *PVWatts Calculator* at National Renewable Energy Laboratory [14] to calculate the hourly solar power. In our simulation, the PV array is fixed on the roof of a household at Maryville, Missouri, with a size of $25\,\mathrm{m}^2$, 20° of the tilt angle and 180° of the azimuth angle. In each round, we randomly pick one day from the whole year.

We run the simulations multiple rounds and then calculate the maximum ratio and the average ratio. The results of the simulations are listed in Table 2.

Table 2. The approximation ratios of algorithms for multiple appliances using Alg-MA.

Simulation #	Mean	Maximum	# of rounds
1st	1.49	2.57	2000
2nd	1.14	2.06	2000

As we can see, the heuristic algorithm does not work very well. The reason is that the schedule of one appliance affects that of another. Scheduling of each appliance only considers minimizing its own cost, and it is likely that shifting some renewable energy from one appliance to another achieves smaller overall total cost.

6 Conclusion

In this paper, we formulate a generic household load scheduling problem which considers both real-time pricing and renewable energy sources. We prove that the household load scheduling problem is NP-hard. We divide household appliances into three categories, and propose algorithms for individual appliances from each category and for multiple appliances from all categories. Specifically, for schedulable and interruptable appliances, such as EWH and AC, we propose a dynamic programming algorithm, which guarantees an optimal or quasi-optimal solution. For scenarios with multiple appliances from all three categories, we propose a heuristic algorithm which schedules appliances in each category in different priority order. We evaluate our algorithms with simulations, validating their efficiency and performance.

References

1. Baughman, M., Siddiqi, S.: Real-time pricing of reactive power: theory and case study results. IEEE Trans. Power Syst. **6**(1), 23–29 (1991)
2. Herter, K.: Residential implementation of critical-peak pricing of electricity. Energy Policy **35**(4), 2121–2130 (2007)
3. Samadi, P., Schober, R., Wong, V.: Optimal energy consumption scheduling using mechanism design for the future smart grid. In: Second IEEE International Conference on Smart Grid Communications (2011)
4. Goudarzi, H., Hatami, S., Pedram, M.: Demand-side load scheduling incentivized by dynamic energy prices. In: Second IEEE International Conference on Smart Grid Communications (2011)
5. Mohsenian-Rad, A., Leon-Garcia, A.: Optimal residential load control with price prediction in real-time electricity pricing environments. IEEE Trans. Smart Grid **1**(2), 120–133 (2010)

6. Du, P., Lu, N.: Appliance commitment for household load scheduling. IEEE Trans. Smart Grid **2**(2), 411–419 (2011)
7. Samadi, P., Mohsenian-Rad, A., Schober, R., Wong, V., Jatskevich, J.: Optimal real-time pricing algorithm based on utility maximization for smart grid. In: First IEEE International Conference on Smart Grid Communications, pp. 415–420 (2010)
8. Lu, J., Sookoor, T., Srinivasan, V., Gao, G., Holben, B., Stankovic, J., Field, E., Whitehouse, K.: The smart thermostat: using occupancy sensors to save energy in homes. In: Proceedings of the 8th ACM Conference on Embedded Networked Sensor Systems (2010)
9. Taylor, Z., Pratt, R.: The effects of model simplifications on equivalent thermal parameters calculated from hourly building performance data. In: Proceedings of the ACEEE Summer Study on Energy Efficiency in Buildings, pp. 10–268 (1988)
10. Pratt, R., Taylor, Z.: Development and testing of an equivalent thermal parameter model of commercial buildings from time-series end-use data. In: Pacific Northwest National Laboratory (PNNL), Richland, WA, Technical report, April 1994
11. Katipamula, S., Chassin, D., Hatley, D., Pratt, R., Hammerstrom, D.: Transactive controls: market-based gridwisetm controls for building systems. Technical report, Pacific Northwest National Laboratory (PNNL), Richland, WA (2006)
12. Ameren real time pricing program. https://www.ameren.com
13. Fairey, P., Parker, D.: A review of hot water draw profiles used in performance analysis of residential domestic hot water systems. Technical report, Florida Solar Energy Center, vol. 15 (2004)
14. National Renewable Energy Laboratory, "PVWatts Calculator". http://pvwatts.nrel.gov/index.php