CSci 243 Graph Practice

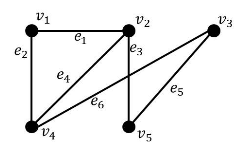
1. Show that a simple graph with $n \geq 2$ vertices must have two vertices of the same degree.

   Case 1: At least one node has $n-1$ edges. If a node has $n-1$ edges, then it is connected to every other vertex, therefore no vertices in the graph have degree 0.

   Case 2: No node has $n-1$ edges.

   In both cases, there are $n-1$ possible degrees for each node $(1,2,\ldots n-1$ or $0,1,\ldots n-2)$, and n nodes. Thus by the Pigeon Hold Principle, at least 2 nodes have the same degree.

2. For the graph below, give its adjacency list, adjacency matrix, and incidence matrix.



Adjacency List

vertex	adjacent vertices
v_1	v_2, v_4
v_2	v_1, v_4, v_5
v_3	v_4, v_5
v_4	v_1, v_2, v_3
v_5	v_2, v_3

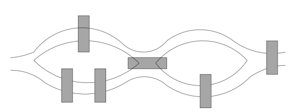
Adjacency Matrix

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Incidence Matrix

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

3. Can someone cross all six bridges shown in the map exactly once and return to the starting point? Why?



   Yes, let the vertices be A (land to the north), B (land to the west), C (land to the east), and D (land to the south). Then $deg(A) = 2$, $deg(B) = 4$, $deg(C) = 2$ and $deg(D) = 4$, and since every degree is even there exists a Euler cycle.

4. Is the graph below undirected or directed? If it's undirected, is it connected? If it's directed, is it strongly connected?

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

   Undirected – asymmetric adjacency matrix. It's strongly connected.