

CSci 243 Homework 2

My name

- (3 points each) Let $P(x)$, $Q(x)$, and $R(x)$ be the statements “ x is a professor”, “ x is ignorant”, and “ x is vain”, respectively. Express each of these statements using quantifiers; logical operations; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists all people.
 - No professor are ignorant.
 - All ignorant people are vain.
 - No professors are vain.
- (2 points each) What are the truth values of these statements?
 - $\exists!xP(x) \rightarrow \exists xP(x)$
 - $\forall xP(x) \rightarrow \exists!xP(x)$
 - $\exists!xP(x) \rightarrow \neg\forall xP(x)$
- Understanding quantified predicates.
 - (3 points) English to quantified predicates: Use predicates, quantifiers, logical and mathematical operators to express statement “There is a positive integer that is not the sum of three squares”.
 - (7 points) Quantified predicate to English: Give the truth value of each of these statement if the domain of all variables consists of all real numbers.
 - $\forall x\exists y(x = y^2)$
 - $\exists x\forall y(xy = 0)$
 - $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$
 - $\exists x\forall y(y \neq 0 \rightarrow xy = 1)$
 - $\forall x\exists y(x + y = 1)$
 - $\forall x\exists y(x + y = 2 \wedge 2x - y = 1)$
 - $\forall x\forall y\exists z(z = (x + y)/2)$
- (2 points each) Rewrite each of these statements so that negations appear only within predicates, i.e., so that no negation is outside a quantifier or an expression involving logical operators.
 - $\neg\exists y\exists xP(x, y)$
 - $\neg\forall x\exists yP(x, y)$
 - $\neg\exists y(Q(y) \wedge \forall x\neg R(x, y))$
 - $\neg\exists y(\exists xR(x, y) \vee \forall xS(x, y))$
 - $\neg\exists y(\forall x\exists zT(x, y, z) \vee \exists x\forall zU(x, y, z))$