CSci 243 Homework 3

My name

1. (10 points) Prove that if n is an integer and $n^3 + 5$ is odd, then n is odd using
   (a) a proof by contraposition.
   (b) a proof by contradiction.
2. (10 points) Prove that n^2 can be divided by 3 if and only if n can be divided by 3.
3. (6 points) Disprove the statement that "Every positive integer can be written as the sum of the squares of three integers" by finding a counterexample.
4. (6 points) Use a proof by cases to show that $min(a, min(b,c)) = min(min(a,b),c)$ whenever a, b, c are real numbers.
5. (6 points) Prove using the notion of without loss of generaility that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.
6. (6 points) A prime number is a positive integer greater than 1 that is only divisible by 1 and itself. Prove or disprove that $\sqrt[k]{p}$ is irrational for any prime number p and any integer $k \geq 2$. Hint: Euclid's Lemma- If a prime divides the *product* of two integers, it must divide at least one of those integers.
7. (6 points) Suppose that a and b are odd integers with $a \neq b$. Show there is a unique integer c such that $|a - c| = |a - b|$.
8. (**Bonus Question** 1 points[1]) Show that $\sqrt{3}$ is irrational.

[1] The points for bonus questions are added directly to the final grade.