## CSci 243 Homework 4

\*\*My name\*\*

- 1. (7 points) Determine whether these statements are true or false.
  - (a)  $\emptyset \in \{\emptyset\}$
  - (b)  $\varnothing \in \{\varnothing, \{\varnothing\}\}$
  - (c)  $\{\emptyset\} \in \{\emptyset\}$
  - $(d) \ \{\varnothing\} \in \{\{\varnothing\}\}$
  - $(e) \ \varnothing \subset \{\varnothing, \{\varnothing\}\}$
  - (f)  $\{\varnothing\} \subset \{\varnothing, \{\varnothing\}\}$
  - $(g) \ \{\{\varnothing\}\} \subset \{\varnothing,\{\varnothing\}\}$
- 2. (4 points) Is each of these sets the power set of a set, where *a* and *b* are distinct elements? If yes, give the original set.
  - (a) Ø
  - (b)  $\{\emptyset, \{a\}\}$
  - (c)  $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
  - (d)  $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

3. (10 points) For sets A, B, and C, prove that  $(B-A) \cup (C-A) = (B \cup C) - A$ 

- (a) by showing each side is a subset of the other side
- (b) by using a membership table
- 4. (5 points) Find these values.
  - (a) [1.1]
  - (b) [1.1]
  - (c) [−0.1]
  - (d) [-0.1]
  - (e)  $\left\lfloor \frac{1}{2} + \left\lceil \frac{1}{2} \right\rceil \right\rfloor$

5. (8 points) Determine whether each of these functions  $f : \mathbb{Z} \to \mathbb{Z}$  is one-to-one, onto, both, or neither.

- (a) f(n) = n+1
- (b)  $f(n) = n^2 + 1$ .
- (c)  $f(n) = n^3$
- (d)  $f(n) = \left\lceil \frac{n}{2} \right\rceil$
- 6. (6 points) Prove that  $\lceil x + n \rceil = \lceil x \rceil + n$ , where x is a real number and n is an integer. (Hint: note the property of ceiling function:  $\lceil x \rceil = n$  if and only if  $n \le x < n + 1$ ).