CSci 243 Homework 4

My name

1. (12 points, 3 points each) For each of these partial sequences of integers, determine the next term of the sequence, and then provide a general formula or rule to generates terms of the sequence.

   (a) $3, 6, 11, 18, 27, 38, 51, 66, 83, 102, \ldots$

   (b) $1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \ldots$

   (c) $0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, \ldots$

   (d) $2, 4, 16, 256, 65536, 4294967296, \ldots$

2. (10 points, 2 points each) Compute each of these sums.

   (a) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i-j)$

   (b) $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i+2j)$

   (c) $\sum_{i=1}^{3} \sum_{j=0}^{2} j$

   (d) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$

   (e) $\sum_{i=99}^{200} (i+1)i(i-1)$

3. (7 points) Find the summation formulae of $\sum_{k=0}^{n}(a+kd)$, i.e., the close-form expression for the summation of general arithmetic progression, by telescoping.

4. (8 points) Find the closed-form expression for $\sum_{k=2}^{n} \frac{1}{k^2-1}$ by telescoping.

5. (8 points) Find the closed-form expressions for the following. Show your work.

   (a) (2 points) $\sum_{i=0}^{n} 3^{i+1} - 3^i$

   (b) (6 points) $\sum_{i=0}^{2n} (-2)^i$ (hint: split series in two parts)

6. (**Bonus Question** 1 points[1]) Let a_n, where $n \geq 0$ be the n_{th} term for the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, ... – constructed by including the integer k exactly k times. Find the closed-form for a_n. You don't need to explain the answer.

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[1]The points for bonus questions are added directly to the final grade.