CSci 243 Homework 6

My name

1. (10 points)

   (a) (4 points) Find the formula for $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n\cdot(n+1)}$ using telescoping.

   (b) (6 points) Prove the formula you found is true by induction.

2. (10 points) Prove by induction that for $n \geq 8$, $n^2 2^n \leq n!$.

3. (10 points) Prove that 21 divides $4^{n+1}+5^{2n-1}$ when n is a positive integer using induction.

4. (10 points) Use strong induction to prove that any positive integer n can be written as a sum of distinct powers of two, that is, as a sum of subset of integers $2^0 = 1$, $2^1 = 2, 2^2 = 4$, and so on. [Hint: consider two cases.]

5. (5 points) Find the flaw with the following proof that $a^n = 1$ for all nonnegative integers n and all nonzero real numbers a.

   Basis step: $a^0 = 1$ is true.

   Inductive step: Assume that $a^j = 1$ for all nonnegative integer $j \leq k$. Consider the case of $k+1$.

   $$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1\cdot 1}{1} = 1.$$

6. (5 points) Find the flaw with the following proof that every postage of 3 cents or more can be formed using just 3-cent and 4-cent stamps.

   Basis step: We can form postage of 3 cents with a single 3-cent stamp and we can form postage of 4 cents using a single 4-cent stamp.

   Inductive step: Assume that we can form postage of j cents for all integers j with $3 \leq j \leq k$ using just 3-cent and 4-cent stamps. We can then form postage of $k+1$ cents by replacing one 3-cent stamp with a 4-cent stamp or by replacing two 4-cent stamps with three 3-cent stamps.