CSci 243 Homework 8

My name

1. A DNA sequence is a string over the alphabet of $\{A, C, G, T\}$, which are called bases. How many 4-element DNA sequences

   (a) (2 points) do not contain the base T?

   (b) (2 points) do not contain the sequence ACG?

   (c) (2 points) contain all four bases, and AC is in the sequence?

   (d) (2 points) begin with A or ends with T?

   (e) (4 points) contain exactly three of the four bases A, T, C, and G?

2. How many functions are there from the set $\{a_1, a_2, ..., a_n\}$, to the set $\{b_1, b_2, ..., b_m\}$, where n and m are positive integers,

   (a) (2 points) that are one-to-one?

   (b) (2 points) that $f(a_1) = b_1$ and $f(a_n) = b_1$?

   (c) (4 points) that $f(a_n) > f(a_1)$?

3. (4 points each) A bowl contains 10 yellow marbles and 10 green marbles. You select marbles at random without looking at them.

   (a) How many marbles must you select to be sure of having at least three marbles of the same color?

   (b) How many marbles must you select to be sure of having at least three green marbles?

4. (8 points) How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \bmod 5 = a_2 \bmod 5$ and $b_1 \bmod 5 = b_2 \bmod 5$?

5. (8 points) How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

6. (6 points) A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

7. (**Bonus Question** 2 points[1]) How many 8-bit strings contain either three consecutive 0's or four consecutive 1's? Explain your solution.

[1] The points for bonus questions are added directly to the final grade.