

## CSci 243 Homework 9

**\*\*My name\*\***

**Note:** Please show steps to explain how you get the answers. For example, answers such as  $C(5, 2) = 10$ , or  $C(5, 2)$ , or descriptive reasoning with your solution is required for full credit. For probability questions, solutions can be left as fractions or decimals to 3 *significant* digits.

- (3 points each) How many bit strings of length 12 contain
  - exactly three 1s?
  - at most three 1s?
  - at least three 1s?
  - an equal number of 0s and 1s?
- (10 points) How many ways are there for 10 women and 6 men to stand in a line so that no two men stand next to each other?
- How many nonnegative integer solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$ , where
  - (2 points)  $x_i > 1$  for all  $i$ ?
  - (2 points)  $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 \geq 5$ , and  $x_6 \geq 6$ ?
  - (3 points)  $x_1 \leq 5$ ?
- How many ways are there to distribute five balls into seven boxes with each box having *at most one ball in it* if
  - (2 points) both balls and boxes are labeled?
  - (2 points) the balls are labeled but the boxes are unlabeled?
  - (2 points) the balls are unlabeled but the boxes are labeled?
  - (2 points) both the balls and boxes are unlabeled?
- (5 points) Using the Binomial Theorem, express  $625x^8 - 1000x^6y^3 + 600x^4y^6 - 160x^2y^9 + 16y^{12}$  in the form  $(a + b)^n$ .
- (7 points) What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?
- On a six-sided die, if rolling a 2 is twice as likely as rolling a 4 and rolling a 4 is twice as likely as rolling each of the other four numbers
  - (4 points) Find the probability of each outcome when the die is rolled.
  - (2 points) What is the probability of rolling an even number?
- (3 points each) Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has
  - exactly three boys?
  - at least one boy?

- (c) at least one girl?
  - (d) all children of the same sex?
9. (8 points) Suppose you select a marble by first picking one of two boxes at random and then selecting a marble from this box at random. The first box contains three green marbles and four yellow marbles, and the second box contains five green marbles and six yellow marbles. What is the probability that you picked a marble from the second box if you have selected a green marble?