

# CSCI 303 Algorithms

## Homework 1

Due: 11:00 in class, September 11, 2001

1. Solve the following recursive functions by iterating:

(a) (4 points) Assume  $n = 2^k$  (i.e.,  $k = \log_2 n$ ) for some  $k$ .

$$f(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2f(\frac{n}{2}) + 1 & \text{if } n \geq 2 \end{cases}$$

(b) (6 points) Assume  $n = 4^k$  (i.e.,  $k = \log_4 n$ ) for some  $k$ .

$$f(n) = \begin{cases} 1 & \text{if } n = 1 \\ 3f(\frac{n}{4}) + n & \text{if } n \geq 2 \end{cases}$$

2. (10 points) Prove by induction the following equation. Make sure you have the three steps in the proof, namely, inductive base, inductive hypothesis, and inductive step.

$$\sum_{i=1}^n i^3 = (\sum_{i=1}^n i)^2$$